ILLUSTRATIVE EXAMPLES

Example 1. *Find the values of*

(*i*) $^{60}P_{48}$ (*ii*) $^{60}P_2$.

Solution. (*i*) $^{60}P_{48} = \dfrac{\lfloor 60}{\lfloor 60-48} = \dfrac{\lfloor 60}{\lfloor 12}$.

(*ii*) $^{60}P_2 = 60 \times 59 = 3540.$

Example 2. *Find n if*

(*i*) $^nP_4 : {}^nP_5 = 1:2$ (*ii*) $P(n, 4) = 20P(n, 2)$.

Solution. (*i*) $^nP_4 : {}^nP_5 = 1:2 \Rightarrow \dfrac{n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)(n-4)} = \dfrac{1}{2}$

$\Rightarrow n - 4 = 2 \Rightarrow n = 6.$

(*ii*) $P(n, 4) = 20\,P(n, 2) \Rightarrow n(n-1)(n-2)(n-3) = 20n(n-1)$

$\Rightarrow (n-2)(n-3) = 20 \Rightarrow (n-2)(n-3) = 5 \times 4$

$\Rightarrow n - 2 = 5 \Rightarrow n = 7$

Example 3. *Find r if:* $^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$.

Solution. Given $^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$

$$\Rightarrow \frac{\lfloor 22}{\lfloor 22-(r+1)} : \frac{\lfloor 20}{\lfloor 20-(r+2)} = 11 : 52$$

$$\Rightarrow \frac{\lfloor 22}{\lfloor 21-r} \times \frac{\lfloor 18-r}{\lfloor 20} = \frac{11}{52}$$

$$\Rightarrow \frac{22.21.\lfloor 20}{(21-r)(20-r)(19-r)\lfloor 18-r} \times \frac{\lfloor 18-r}{\lfloor 20} = \frac{11}{52}$$

$$\Rightarrow \frac{22.21}{(21-r)(20-r)(19-r)} = \frac{11}{52}$$

$\Rightarrow (21-r)(20-r)(19-r) = 2 \times 21 \times 52$

$\Rightarrow (21-r)(20-r)(19-r) = 14.13.12$

$\Rightarrow 21 - r = 14$ (by inspection)

$\Rightarrow r = 7.$

Example 4. *Prove that*

(*i*) $P(n, r) = n.P(n-1, r-1)$ (*iii*) ${}^nP_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$.

Solution. (*i*) R.H.S. $= n.P(n-1, r-1) = n.\dfrac{\lfloor n-1}{\lfloor n-1-(r-1)} = \dfrac{n.\lfloor n-1}{\lfloor n-r} = \dfrac{\lfloor n}{\lfloor n-r}$

$= {}^nP_r$.

(*ii*) R.H.S. $= {}^{n-1}P_r + r.{}^{n-1}P_{r-1} = \dfrac{\lfloor n-1}{\lfloor n-1-r} + r.\dfrac{\lfloor n-1}{\lfloor n-1-(r-1)}$

$$= \frac{\lfloor n-1}{\lfloor n-r-1} + r.\frac{\lfloor n-1}{\lfloor n-r} = \frac{\lfloor n-1}{\lfloor n-r-1} + \frac{r\lfloor n-1}{(n-r)\lfloor n-r-1}$$

$$= \frac{\lfloor n-1}{\lfloor n-r-1}\left(1+\frac{r}{n-r}\right) = \frac{\lfloor n-1}{\lfloor n-r-1}.\frac{n-r+r}{(n-r)}$$

$$= \frac{n\lfloor n-1}{(n-r)\lfloor n-r-1} = \frac{\lfloor n}{\lfloor n-r} = {}^nP_r.$$

Example 5. *A customer forgets a four-digit code for an Automatic Teller Machine (ATM) in a bank. However, he remembers that this code consists of 3, 4, 7 and 9. Find the largest possible number of trials necessary to obtain the correct code.*

Solution. As the code is four-digit and there are four different digits 3, 4, 7 and 9, all digits are to be used, all of them can be arranged in ${}^4P_4$ ways.

Hence, the largest number of trials necessary to obtain the correct code

$= {}^4P_4 = \lfloor 4 = 4 \times 3 \times 2 \times 1 = 24$.

Example 6. *How many 4-digit numbers can be formed by using the digits 1 to 9 if repetition of digits is not allowed?*

Solution. As there are 9 different digits (1 to 9) and repetition of digits is not allowed, any four of them can be arranged in ${}^9P_4$ ways.

Hence, the required number of 4-digit numbers $= {}^9P_4 = 9 \times 8 \times 7 \times 6 = 3024$.

Note

We know that any 4-digit number cannot have 0 in the first place (thousand's place). In the above problem, none of the given digits is 0. However, if 0 is among the given digits then the problem would be different.

Example 7. *Ten students participate in a debate. In how many ways can the first three prizes be won?*

Solution. The given problem is similar to one of filling 3 places with any 3 of 10 distinct objects, repetitions not allowed. This can be done in ${}^{10}P_3$ ways.

Hence, required number of winning the prizes $= {}^{10}P_3 = 10 \times 9 \times 8 = 720$.

Example 8. *In how many ways can three sports prizes be given to 20 boys when a boy may receive any number of prizes?*

Solution. The first prize can be given in 20 ways, and then second prize can also be given in 20 ways, since repetition is allowed. Similarly, third prize can also be given in 20 ways. Thus by principle of counting, required number of ways $= 20 \times 20 \times 20 = 8000$.

Alternatively

As repetition is allowed, the required number of ways of receiving prizes

$= 20^3$ (Theorem 2)

$= 20 \times 20 \times 20 = 8000$.

Example 9. *Find the total number of ways in which n distinct balls can be put into two different boxes so that no box remains empty.*

Solution. Let the two different boxes be B_1 and B_2.

Each ball can be dealt with two ways—it can be put either in box B_1 or in box B_2. As there are n distinct balls, the total number of ways putting these balls in two boxes = 2^n.

These number of ways also include the ways in which all balls are put in box B_1 or in box B_2 *i.e.* either the box B_2 is empty or the box B_1 is empty.

$\therefore$ The number of ways of putting n distinct balls in two different boxes so that no box remains empty = $2^n - 2$.

Example 10. *To attend a joint Summit, two countries send 5 delegates each to a negotiating table. A rectangular table is used with 5 chairs on each side. If each country is assigned a long side of the table, how many seating arrangements are possible?*

Solution. On one side of the long table, 5 delegates of one country on 5 chairs can be seated in ${}^5P_5$ *i.e.* $\underline{|5}$ ways.

Similarly, on the other side of the long table, 5 delegates of other country on 5 chairs can be seated in $\underline{|5}$ ways.

The two sides of the long table can be assigned to the countries in 2 ways.

Hence, number of seating arrangements = $\underline{|5} \times \underline{|5} \times 2$

$$= 120 \times 120 \times 2 = 28800.$$

Example 11. *There are 5 true-false questions in a test. If no two students have answered the same sequence of answers and no student has given all correct answers. how many students are there in the class for this to happen?*

Solution. Each question can be attempted in two ways — may write true or may write false.

$\therefore$ The total number of ways of attempting 5 questions = $2^5 = 32$.

These number of ways also include the way in which all answers are correct.

$\therefore$ Total number of ways = $32 - 1 = 31$.

Hence, there are 31 students in the class.

Example 12. *A question paper contains 10 true-false questions. In how many ways can a student attempt one or more questions?*

Solution. Each question can be attempted in three ways—may not be answered, may write true or may write false.

$\therefore$ The total number of ways of attempting 10 questions = 3^{10}.

These number of ways also include the way in which no question is attempted.

$\therefore$ The number of ways of attempting one or more questions = $3^{10} - 1$.

Example 13. *Find the number of ways of selecting a president, a secretary and a treasurer from a board of 8 members, if 2 members who were holding one of these posts cannot be nominated again.*

Solution. Given that there are total 8 members in a board out of which 2 members who were already holding one of these posts cannot be nominated again. So, we are left with 8 – 2 *i.e.* 6 members.

Now number of ways of selecting a president, a secretary and a treasurer from among 6 members is equal to number of arrangements of 6 members taken 3 at a time *i.e.* ${}^6P_3 = \frac{\underline{|6}}{\underline{|3}} = 120$.

Example 14. *Find the number of signals which can be given with 5 flags of different colours hoisted one above the other, by using any number of flags.*

Solution. When 1 flag is used, number of signals = $^5P_1 = 5$.

When 2 flags are used, number of signals = $^5P_2 = 5 \times 4 = 20$.

When 3 flags are used, number of signals = $^5P_3 = 5 \times 4 \times 3 = 60$.

When 4 flags are used, number of signals = $^5P_4 = 5 \times 4 \times 3 \times 2 = 120$.

When 5 flags are used, number of signals = $^5P_5 = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Hence, the total number of signals = $5 + 20 + 60 + 120 + 120 = 325$.

Example 15. *How many words can be made from the letters in the word MONDAY, assuming that no letter is repeated, if*

(*i*) *4 letters are used at a time?* (*ii*) *all letters are used at a time?*

Solution. The given word MONDAY, has 6 different letters out of which 2 are vowels and 4 are consonants. Repetition of letters is not allowed in any word.

(*i*) Required number of words = $^6P_4 = 6 \times 5 \times 4 \times 3 = 360$.

(*ii*) Required number of words = $^6P_6 = \underline{|6} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

EXERCISE 7.3

1. Find the values of

(*i*) $P(4, 3)$ (*ii*) $P(65, 2)$ (*iii*) $^7P_5$ (*iv*) $^{10}P_3$.

2. Prove that

(*i*) $^{10}P_3 = {^9P_3} + 3\,{^9P_2}$ (*ii*) $^nP_n = 2\,{^nP_{n-2}}$ (*iii*) $P(n, n) = P(n, n-1)$.

3. Find the value of n if

(*i*) $^{n+1}P_3 = {^nP_4}$ (*ii*) $5P(4, n) = 6P(5, n-1)$

(*iii*) $P(n-1, 3) : P(n+1, 3) = 5 : 12$.

4. Six candidates are called for interview to fill some vacant posts in an office. Assuming that each candidate is fit for each post, determine the number of ways in which

(*i*) two posts can be filled (*ii*) three posts can be filled.

5. How many 3-digit numbers can be formed by using the digits 1 to 9 if

(*i*) no digit is repeated? (*ii*) digits can be repeated?

6. (*i*) In how many ways can 6 women draw water from 6 taps if no tap remains unused?

(*ii*) Seven candidates are contesting an election. In how many ways can their names be listed on the ballot paper?

7. (*i*) From a committee of 8 persons, in how many ways can you choose a chairman and a vice chairman assuming that one cannot hold more than one position.

(*ii*) From 12 persons, in how many ways can we choose a president, vice-president, secretary and a treasurer assuming that each of the 12 persons can hold any office and no person can hold more than one office?

8. (*i*) A man has 6 friends. In how many ways can he invite one or more of his friends at a dinner?

(*ii*) There are 8 true-false statements in a question paper. How many sequences of answers are possible?

9. In how many ways three different rings can be worn in four fingers with atmost one in each finger?

10. Find the value of r if

(*i*) $^5P_r = 2\,{^6P_{r-1}}$ (*ii*) $P(11, r) = P(12, r-1)$

(*iii*) $P(15, r-1) : P(16, r-1) = 3 : 4$.

11. How many words can be formed using all letters (using each letter exactly once) of the following words:

(i) DELHI (ii) JAIPUR (iii) HEXAGON (iv) EQUATION

Answers

1. (i) 24	(ii) 4160	(iii) 2520	(iv) 720		
3. (i) 5	(ii) 3	(iii) 8	**4.** (i) 30	(ii) 120	
5. (i) 504	(ii) 729	**6.** (i) 720	(ii) 5040	**7.** (i) 56	(ii) 11880
8. (i) 63	(ii) 256	**9.** 24	**10.** (i) 3	(ii) 9	(iii) 5
11. (i) 120	(ii) 720	(iii) 5040	(iv) 40320		