AXIOMATIC APPROACH TO PROBABILITY

Let S be the sample space of a random experiment, then probability P is a real valued function whose domain is the power set of S *i.e.* P(S) and range is the closed interval [0, 1] *i.e.*

$$P : P(S) \to [0, 1]$$

satisfying the following axioms :

(*i*) For any event A, $0 \le P(A) \le 1$

(*ii*) $P(S) = 1$

(*iii*) If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B).$$

From (*iii*), it follows that $P(\phi) = 0$.

To prove this, we take $B = \phi$ and note that A and ϕ are disjoint events *i.e.* mutually exclusive events. Therefore, by axiom (*iii*), we have

$$P(A \cup \phi) = P(A) + P(\phi) \text{ but } A \cup \phi = A$$

$$\Rightarrow \quad P(A) = P(A) + P(\phi) \Rightarrow P(\phi) = 0.$$

If the sample space contains n (>1) elementary events *i.e.* outcomes $w_1, w_2, w_3, \ldots, w_n$ *i.e.* $S = \{w_1, w_2, w_3, \ldots, w_n\}$, then from the axiomatic definition of probability, we have :

(*i*) $0 < P(w_i) < 1$ for each $w_i \in S$

(*ii*) $P(w_1) + P(w_2) + P(w_3) + \ldots + P(w_n) = 1$

(*iii*) For any event A, $P(A) = \Sigma P(w_i)$, for all $w_i \in A$.

Probabilities of equally likely outcomes

Let a sample space of a random experiment be $S = \{w_1, w_2, \ldots, w_n\}$, $n > 1$.

If all the outcomes are equally likely to occur, then the chance of occurrence of each simple event must be the same.

Let $P(w_i) = p$ for all $w_i \in S$, where $0 < p < 1$.

Since $\sum_{i=1}^{n} P(w_i) = 1$,

we have $\sum_{i=1}^{n} p = 1$ *i.e.* $p + p + p \ldots (n \text{ times}) = 1$

$$\Rightarrow np = 1 \text{ or } p = \frac{1}{n}.$$

Let S be the sample space associated with a random experiment and E be an event such that number of elements in set S is n *i.e.* $O(S) = n$, and number of elements in set E is m *i.e.* $O(E) = m$, then in case of equally likely outcomes,

$$P(E) = \frac{m}{n} = \frac{\text{number of outcomes favourable to E}}{\text{total number of possible outcomes}}.$$

For example, if a die is rolled, then S = {1, 2, 3, 4, 5, 6}
and if A = {getting an odd number}, then A = {1, 3, 5},

$\therefore$ The probability of getting an odd number

$$= P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

Similarly, if B = {getting a number less than 5}, then B = {1, 2, 3, 4}.

$\therefore$ The probability of getting a number less than 5

$$= P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}.$$

Odds in favour of and odds against an event

Let S be the sample space of a random experiment and all the outcomes are equally likely.

If an event E can happen in 'a' ways and fail in 'b' ways, then odds in favour of E are a : b, odds against E are b : a, and probability of E is

$$P(E) = \frac{a}{a+b}.$$

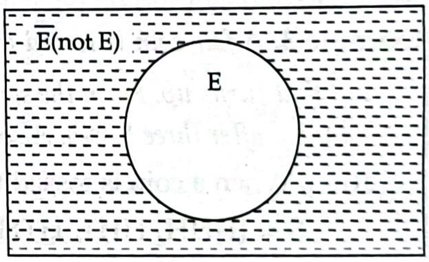
If the probability of event E is given to be p, then

$$\text{odds in favour of event } E = \frac{p}{1-p} \text{ i.e. } p : (1-p),$$

$$\text{and odds against event } E = \frac{1-p}{p} \text{ i.e. } (1-p) : p.$$

Probability of event 'not E'

If E is any event, then P(not E) = 1 – P(E).



Proof. Let S be the sample space of a random experiment and E be any event associated with the random experiment, then 'not E' (also written as $\bar{E}$, E' or E^c) is also an event such that $E \cap \bar{E} = \phi$ *i.e.* E and $\bar{E}$ are mutually disjoint and $E \cup \bar{E} = S$. By axioms (*iii*) and (*ii*), we have :

$$P(E \cup \bar{E}) = P(E) + P(\bar{E}) \text{ and } P(E \cup \bar{E}) = P(S) = 1$$

$$\Rightarrow P(E) + P(\bar{E}) = 1 \Rightarrow P(\bar{E}) = 1 - P(E)$$

$$\Rightarrow P(\text{not } E) = 1 - P(E).$$

ILLUSTRATIVE EXAMPLES

Example 1. *An experiment has four possible outcomes A, B, C and D. Explain why the following assignments of probabilities are not permissible :*

(*i*) $P(A) = 0.12, P(B) = 0.63, P(C) = 0.45, P(D) = -0.20$

(*ii*) $P(A) = \frac{9}{120}, P(B) = \frac{45}{120}, P(C) = \frac{27}{120}, P(D) = \frac{46}{120}$

Solution. (*i*) Here P(D) = –0.20, which is negative. This is not permissible as $0 < P(w_i) < 1$.

(ii) If S is the sample space, then

$$P(S) = P(A) + P(B) + P(C) + P(D) = \frac{9}{120} + \frac{45}{120} + \frac{27}{120} + \frac{46}{120} = \frac{127}{120} \neq 1.$$

Hence, the given assignment of probabilities is not permissible.

Example 2. *In an experiment of tossing a coin twice, describe the sample space. What is the probability of getting*

(i) *no heads* (ii) *exactly one head* (iii) *two heads*

(iv) *atleast one head* (v) *one head and one tail* (vi) *atleast one tail?*

Solution. Sample space is {HH, HT, TH, TT}. It has 4 outcomes which are equally likely.

(i) The outcome favourable to the event "no heads" is TT.

$$\therefore \quad P(\text{no heads}) = \frac{1}{4}.$$

(ii) The outcomes favourable to the event "exactly one head" are HT, TH.

$$\therefore \quad P(\text{exactly one head}) = \frac{2}{4} = \frac{1}{2}.$$

(iii) The outcome favourable to the event "two heads" is HH.

$$\therefore \quad P(\text{two heads}) = \frac{1}{4}.$$

(iv) The outcomes favourable to the event "atleast one head" are HH, HT and TH.

$$\therefore \quad P(\text{atleast one head}) = \frac{3}{4}.$$

(v) The outcomes favourable to the event "one head and one tail" are HT, TH.

$$\therefore \quad P(\text{one head and one tail}) = \frac{2}{4} = \frac{1}{2}.$$

(vi) The outcomes favourable to the event "atleast one tail" are TT, TH, HT.

$$\therefore \quad P(\text{atleast one tail}) = \frac{3}{4}.$$

Example 3. *A fair coin is tossed three times, and a person wins ₹1 for each head and loses ₹1.50 for each tail that turns up. Form the sample space and calculate how many different amounts of money you can win/lose after three tosses. Also find the probability of having each of these amounts.*

Solution. When a coin is tossed three times, then the sample space of the experiment is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

After 3 tosses, the different possibilities are :

Case I. You may get 'three heads', then gain = ₹(3 × 1) = ₹3.

Case II. You may get '2 heads and one tail', then

gain = ₹(2 × 1 + (−1.5) × 1) = ₹0.50

Case III. You may get 'one head and 2 tails', then

loss = ₹(2 × 1.50 −1 × 1) = ₹2.

Case IV. You may get 'three tails', then

loss = ₹(3 × 1.50) = ₹4.50.

Let A, B, C and D be the events of getting 'three heads', '2 heads and one tail', 'one heads and 2 tails' and three tails' respectively. Then

A = {HHH}, B = {HHT, HTH, THH}

C = {HTT, THT, TTH} and

D = {TTT}

$\therefore$ P (winning ₹3) = P(A) = $\frac{1}{8}$, P (winning ₹0.50) = P(B) = $\frac{3}{8}$,

P (losing ₹2) = P(C) = $\frac{3}{8}$ and P (losing ₹4.50) = P(D) = $\frac{1}{8}$.

Example 4. *In a single throw of two dice, what is the probability of getting*

(*i*) *A total of 9?* (*ii*) *two aces?*

(*iii*) *atleast one ace?* (*iv*) *doublets?*

(*v*) *five-six (i.e. one die comes up with five and the other with six)?*

(*vi*) *a multiple of 2 on one die and a multiple of 3 on the other?*

(*vii*) *a sum of 9 or 11?* (*viii*) *the sum as a prime number?*

(*ix*) *the sum is atleast 10?* (*x*) *a total of atmost 9?*

Solution. Let S be the sample space of the given experiment, then

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

It has 36 outcomes which are equally likely.

(*i*) The outcomes favourable to the event "a total of 9" are (3, 6), (4, 5), (5, 4), (6,3).

$\therefore$ P(a total of 9) = $\frac{4}{36} = \frac{1}{9}$.

(*ii*) The outcome favourable to the event "two aces" is (1, 1) only.

$\therefore$ P(two aces) = $\frac{1}{36}$.

(*iii*) The outcomes favourable to the event "atleast one ace" are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1). The number of favourable outcomes = 11.

$\therefore$ P(atleast one ace) = $\frac{11}{36}$.

(*iv*) A 'doublet' means that both dice come up with same number. So favourable outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6).

$\therefore$ P(doublets) = $\frac{6}{36} = \frac{1}{6}$.

(*v*) The outcomes favourable to the event 'five-six' are (5, 6), (6, 5).

$\therefore$ P(five-six) = $\frac{2}{36} = \frac{1}{18}$.

(*vi*) Outcomes favourable to the event "a multiple of 2 on one die and a multiple of 3 on other" are (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2), (6, 4).

The number of favourable outcomes = 11.

$\therefore$ Required probability = $\frac{11}{36}$.

(*vii*) The outcomes favourable to the event 'a sum of 9 or 11' are

(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5).

The number of favourable outcomes = 6.

$\therefore$ P(a sum of 9 or 11) = $\frac{6}{36} = \frac{1}{6}$.

(*viii*) Outcomes favourable to the event 'sum as a prime number *i.e.* the sum as 2, 3, 5, 7, 11' are

(1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6), (6, 5).

The number of favourable outcomes = 15.

$\therefore$ P(sum as a prime number) = $\frac{15}{36} = \frac{5}{12}$.

(*ix*) The outcomes favourable to the event 'sum is atleast 10 *i.e.* the sum is 10, 11, 12' are (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6).

The number of favourable outcomes = 6.

$\therefore$ P(sum of atleast 10) = $\frac{6}{36} = \frac{1}{6}$.

(*x*) Let A be the event 'the sum is atmost 9' then $\bar{A}$ denotes the event 'the sum is greater than 9 *i.e.* the sum is 10, 11, 12'. The outcomes favourable to the event $\bar{A}$ are (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6).

The number of outcomes favourable to the event $\bar{A}$ = 6.

$\therefore$ $P(\bar{A}) = \frac{6}{36} = \frac{1}{6}$.

$\therefore$ $P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$.

Example 5. *Three dice are thrown simultaneously. Find the probability of getting a total of atleast 6.*

Solution. When three dice are thrown simultaneously, the total number of outcomes = $6 \times 6 \times 6 = 216$. All the outcomes are equally likely. Let A be the event of getting a total of atleast 6 *i.e.* total ≥ 6, then $\bar{A}$ denotes the event of getting a total of less than 6 *i.e.* a total of 3, 4, 5.

$\therefore$ $\bar{A}$ = {(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)}.

The number of outcomes favourable to the event $\bar{A}$ = 10.

$\therefore$ $P(\bar{A}) = \frac{10}{216} = \frac{5}{108}$.

$\therefore$ $P(A) = 1 - P(\bar{A}) = 1 - \frac{5}{108} = \frac{103}{108}$.

Example 6. *From the set {1, 2, 3, 4, 5}, two numbers a and b ($a \neq b$) are chosen at random. Find the probability that $\frac{a}{b}$ is an integer.*

Solution. Let A { 1, 2, 3, 4, 5} and S be the sample space of random experiment, then

$S = \{(a, b) : a, b \in A, a \neq b\}$

= {(1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 3), (2, 4), (2, 5), (3, 1), (3, 2), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4)}

The sample space S has 20 equally likely outcomes.

Let E be the event '$\frac{a}{b}$ is an integer'. Then

E = {(2, 1), (3, 1), (4, 1), (4, 2), (5, 1)}, it has 5 outcomes.

$\therefore$ Required probability = P(E) = $\frac{5}{20} = \frac{1}{4}$.

Example 7. *A ball is drawn from a bag containing 5 white and 7 black balls.*

(*i*) *What is the probability of drawing a white ball?*

(*ii*) *What are the odds against drawing a white ball?*

(*iii*) *If two balls are drawn simultaneously, what is the probability that both balls are white?*

(*iv*) *What are the odds in favour of drawing two black balls?*

(*v*) *What is the probability of drawing one white and one black ball?*

Solution. (*i*) Since there are 5 white out of a total of 12 balls, the probability of drawing a white ball is $\frac{5}{12}$.

(*ii*) The odds against drawing a white ball are $(1-p):p$ *i.e.*

$$\left(1-\frac{5}{12}\right):\frac{5}{12}=\frac{7}{12}:\frac{5}{12}=7:5.$$

(*iii*) Now if two balls are to be drawn simultaneously, the total number of ways of doing so is $^{12}C_2$, and number of ways of choosing two white balls is $^5C_2$; so the probability of drawing two white balls

$$=\frac{^5C_2}{^{12}C_2}=\frac{5\times4}{1\times2}\times\frac{1\times2}{12\times11}=\frac{5}{33}.$$

(*iv*) The probability of drawing two black balls $=\frac{^7C_2}{^{12}C_2}=\frac{7\times6}{1\times2}\times\frac{1\times2}{12\times11}=\frac{7}{22}$.

So the odds in favour of drawing two black balls are

$$p:(1-p)=\frac{7}{22}:\left(1-\frac{7}{22}\right)=\frac{7}{22}:\frac{15}{22}=7:15.$$

(*v*) The number of ways of choosing one white ball and one black ball is $^5C_1\times{}^7C_1=5\times7=35$. So the probability of drawing one white and one black ball

$$=\frac{35}{^{12}C_2}=\frac{35\times1\times2}{12\times11}=\frac{35}{66}.$$

Example 8. *A committee of 5 persons is to be constituted from a group of 6 gents and 8 ladies. If the selection is made randomly, find the probability that there are 3 ladies and 2 gents in the committee.*

Solution. 5 persons out of a total of 14 can be selected in $^{14}C_5$ ways. 3 ladies and 2 gents can be chosen out of 8 ladies and 6 gents in $^8C_3\times{}^6C_2$ ways.

$$\therefore \quad \text{Required probability} = \frac{^8C_3\times{}^6C_2}{^{14}C_5}$$

$$=\frac{8.7.6}{1.2.3}\times\frac{6.5}{1.2}\times\frac{1.2.3.4.5}{14.13.12.11.10}=\frac{60}{143}.$$

Example 9. *A bag contains 30 tickets, numbered 1 to 30. Five tickets are drawn at random and arranged in ascending order. Find the probability that the third number is 20.*

Solution. Total number of cases $={}^{30}C_5$.

If five tickets are arranged in ascending order and third ticket is 20, then tickets are $a_1, a_2, 20, a_4, a_5$, where $a_1, a_2\in\{1,2,\ldots,19\}$ and $a_4, a_5\in\{21,22,\ldots,30\}$.

Number of favourable cases $={}^{19}C_2\times{}^{10}C_2$

$$\therefore \text{Required probability}=\frac{^{19}C_2\times{}^{10}C_2}{^{30}C_5}=\frac{19.18\times10.9}{1.2\times1.2}\times\frac{1.2.3.4.5}{30.29.28.27.26}=\frac{285}{5278}.$$

Example 10. *A box contains 9 red balls, 5 blue balls and 6 green balls. Three balls are drawn from the box at random. Find the probability that*

(*i*) *all will be blue* (*ii*) *atleast one will be green.*

Solution. Total number of balls $=9+5+6=20$.

3 balls can be drawn in $^{20}C_3$ ways.

(i) Out of 5 blue balls, 3 blue balls can be drawn in ${}^5C_3$ ways.

$$\therefore \quad P(\text{all blue balls}) = \frac{{}^5C_3}{{}^{20}C_3} = \frac{5.4.3}{1.2.3} \times \frac{1.2.3}{20.19.18} = \frac{1}{114}.$$

(ii) Let A be event atleast one ball is green, then $\bar{A}$ is event that no ball is green.

Number of non-green balls = 9 + 5 = 14.

$\therefore$ 3 non-green balls can be drawn in ${}^{14}C_3$ ways

$$\therefore \quad P(\bar{A}) = \frac{{}^{14}C_3}{{}^{20}C_3} = \frac{14.13.12}{1.2.3} \times \frac{1.2.3}{20.19.18} = \frac{91}{285}.$$

$$\therefore \quad P(A) = 1 - P(\bar{A}) = 1 - \frac{91}{285} = \frac{194}{285}.$$

Example 11. *Four cards are drawn at random from a pack of 52 playing cards. Find the probability of getting*

(i) *all the four cards of the same suit*
(ii) *all the four cards of the same denomination*
(iii) *one card from each suit*
(iv) *all four picture cards*
(v) *two red cards and two black cards*
(vi) *all cards of the same colour*
(vii) *getting four aces.*

Solution. (i) There are ${}^{13}C_4$ ways of choosing 4 spades; ditto for all other three suits.

$\therefore$ Number of ways of choosing 4 cards of same suit = $4.{}^{13}C_4$

Total number of ways of choosing 4 cards from 52 cards is ${}^{52}C_4$

$$\therefore \quad P(\text{all four cards of same suit}) = \frac{4.{}^{13}C_4}{{}^{52}C_4}$$

$$= 4.\frac{13.12.11.10}{1.2.3.4} \times \frac{1.2.3.4}{52.51.50.49} = \frac{44}{4165}.$$

(ii) There are only 13 ways of choosing all four cards of same denomination – four aces, four kings, ... etc.

$$\therefore \quad P(\text{all four cards of same denomination}) = \frac{13}{{}^{52}C_4} = \frac{13}{270725} = \frac{1}{20825}.$$

(iii) There are $({}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1)$ ways of choosing one card from each suit. Hence

$$P(\text{one card from each suit}) = \frac{({}^{13}C_1)^4}{{}^{52}C_4} = \frac{13.13.13.13 \times 1.2.3.4}{52.51.50.49} = \frac{2197}{20825}.$$

(iv) There are 12 picture cards. There are ${}^{12}C_4$ ways of choosing four picture cards.

$$\text{So } P(\text{four picture cards}) = \frac{{}^{12}C_4}{{}^{52}C_4} = \frac{12.11.10.9}{52.51.50.49} = \frac{99}{54145}.$$

(v) There are 26 red cards and 26 black cards. So there are $({}^{26}C_2 \times {}^{26}C_2)$ ways of choosing two red and two black cards.

$$\therefore \quad P(\text{two red and two black cards}) = \frac{{}^{26}C_2 \times {}^{26}C_2}{{}^{52}C_4} = \frac{325}{833}.$$

(vi) There are ${}^{26}C_4$ ways of getting 4 red cards and ${}^{26}C_4$ ways of getting black cards.

$$\therefore \quad P(\text{all cards of same colour}) = \frac{2.{}^{26}C_4}{{}^{52}C_4} = \frac{92}{833}.$$

(vii) $P(\text{getting four aces}) = \dfrac{{}^4C_4}{{}^{52}C_4} = \dfrac{1}{270725}.$

Example 12. *Find the probability that when a hand of 7 cards is drawn from a well-shuffled deck of 52 cards, it contains*

(*i*) *all kings* (*ii*) *exactly 3 kings* (*iii*) *atleast 3 kings.*

Solution. Total number of ways of choosing 7 cards from 52 cards = ${}^{52}C_7$.

(*i*) As the hand of 7 cards contains all kings, so we choose 4 kings out of 4 kings and 3 other cards out of the remaining 48 cards.

The number of ways of choosing 7 cards containing all kings

$$= {}^4C_4 \times {}^{48}C_3.$$

$$\therefore \text{ Required probability } = \frac{{}^4C_4 \times {}^{48}C_3}{{}^{52}C_7} = \frac{1.48.47.46}{1.2.3} \times \frac{1.2.3.4.5.6.7}{52.51.50.49.48.47.46}$$

$$= \frac{4.5.6.7}{52.51.50.49} = \frac{1}{7735}.$$

(*ii*) As the hand of 7 cards contains 3 kings, so we are to choose 3 kings out of 4 kings and 4 other cards out of remaining 48 cards.

The number of ways of choosing 7 cards containing exactly 3 kings

$$= {}^4C_3 \times {}^{48}C_4.$$

$$\therefore \text{ Required probability} = \frac{{}^4C_3 \times {}^{48}C_4}{{}^{52}C_7} = \frac{4.48.47.46.45}{1.2.3.4} \times \frac{1.2.3.4.5.6.7}{52.51.50.49.48.47.46}$$

$$= \frac{4.45.5.6.7}{52.51.50.49} = \frac{9}{1547}.$$

(*iii*) A hand of 7 cards containing atleast 3 kings means that it contains 3 kings or 4 kings.

The number of ways of choosing 7 cards containing 3 kings and 4 other cards or choosing 4 kings and 3 other cards = ${}^4C_3 \times {}^{48}C_4 + {}^4C_4 \times {}^{48}C_3$.

$$\therefore \text{ Required probability } = \frac{{}^4C_3 \times {}^{48}C_4 + {}^4C_4 \times {}^{48}C_3}{{}^{52}C_7} = \frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}.$$

Example 13. *If the letters of the word ALGORITHM are arranged at random in a row, then what is the probability that the letters GOR must remain as a unit?*

Solution. The word 'ALGORITHM' has 9 different letters.

The number of ways of arranging these letters at random in a row = $\underline{|9}$.

Now let us consider those arrangement in which the letters GOR remain together as a unit. So, we have to arrange six letters A, L, I, T, H, M and a pack of three letters (GOR), which can be arranged in $\underline{|7}$ ways.

$$\therefore \text{ Required probability} = \frac{\underline{|7}}{\underline{|9}} = \frac{\underline{|7}}{9 \times 8 \times \underline{|7}} = \frac{1}{72}.$$

Example 14. *In a relay race there are five teams A, B, C, D and E.*

(*i*) *What is the probability that A, B and C finish first, second and third respectively?*

(*ii*) *What is the probability that A, B and C are first three to finish (in any order)?*

Solution. We assume that all finishing orders are equally likely. If we consider sample space consisting of all finishing orders in the first three places, we will have ${}^5P_3$

i.e. $\dfrac{\underline{|5}}{\underline{|5-3}} = 5 \times 4 \times 3 = 60$ sample points, each with a probability of $\dfrac{1}{60}$.

(i) A, B and C finish first, second and third respectively. There is only one finishing order for this *i.e.* ABC.

$\therefore$ The required probability $= \frac{1}{60}$.

(ii) Out of 60 sample points, there are $\lfloor 3 = 6$ arrangements which contain A, B, C (in any order). These are ABC, ACB, BAC, BCA, CAB, CBA.

$\therefore$ The required probability $= \frac{6}{60} = \frac{1}{10}$.

Example 15. *The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e. from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?*

Solution. The given number lock has 4 wheels, each labelled with ten digits *i.e.* from 0 to 9.

The total number of sequences of 4 digits with no repeats

$= 10 \times 9 \times 8 \times 7 = 5040.$

$\square\ \square\ \square\ \square$

10 9 8 7

As there is only one correct sequence of four digits to open the lock,

required probability $= \frac{1}{5040}$.

Example 16. *Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that*

(i) *you both enter the same section?*

(ii) *you both enter the different sections?*

Solution. The total number of ways of forming two sections of 40 and 60 students out of 100 students $= {}^{100}C_{40}$ (because if we form a section of 40 students out of 100 students, then another section of 60 students is automatically formed).

(i) If you and your friend enter in the section of 40 students, then we are to choose 38 more students out of the remaining 98 students, so the number of ways of forming this section $= {}^{98}C_{38}$.

Similarly, if you and your friend enter in the section of 60 students, then the number of ways of forming this section $= {}^{98}C_{58}$.

Therefore, the number of ways of forming two sections of 40 and 60 students so that you and your friend enter the same section $= {}^{98}C_{38} + {}^{98}C_{58}$.

$$\therefore \text{ Required probability } = \frac{{}^{98}C_{38} + {}^{98}C_{58}}{{}^{100}C_{40}} = \frac{{}^{98}C_{38}}{{}^{100}C_{40}} + \frac{{}^{98}C_{58}}{{}^{100}C_{40}}$$

$$= \frac{\lfloor 98}{\lfloor 38 \lfloor 60} \times \frac{\lfloor 40 \lfloor 60}{\lfloor 100} + \frac{\lfloor 98}{\lfloor 58 \lfloor 40} \times \frac{\lfloor 40 \lfloor 60}{\lfloor 100}$$

$$= \frac{40.39}{100.99} + \frac{60.59}{100.99} = \frac{26}{165} + \frac{59}{165} = \frac{85}{165} = \frac{17}{33}.$$

(ii) Probability that you and your friend both enter the different sections

$= 1 -$ probability that you both enter the same section

$= 1 - \frac{17}{33} = \frac{16}{33}$.

EXERCISE 12.3

1. A die is thrown once. What is the probability of getting
   (*i*) a prime number
   (*ii*) a number greater than or equal to 3
   (*iii*) a number less than or equal to one
   (*iv*) a number greater than 6
   (*v*) a number less than 6?
2. A card is drawn randomly from a well-shuffled pack of 52 playing cards. What is the probability of drawing
   (*i*) a diamond
   (*ii*) not a diamond
   (*iii*) an ace
   (*iv*) not an ace
   (*v*) an ace of spades
   (*vi*) a black card
   (*vii*) not a black card
   (*viii*) a ten or a spade
   (*ix*) a picture card or a spade?
   (*x*) a king or 9 of hearts or 3 of spades?
3. A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. If a disc is drawn from the bag, then what is probability that it is
   (*i*) red
   (*ii*) yellow
   (*iii*) blue
   (*iv*) not blue
   (*v*) either red or yellow?
4. A letter is chosen at random from the word 'ASSASSINATION'. What is the probability that it is a
   (*i*) vowel?
   (*ii*) consonant?
5. A bag contains twenty discs numbered 1 to 20. A disc is drawn from the bag. What is the probability that the number on it
   (*i*) is a multiple of 3
   (*ii*) is a prime number
   (*iii*) contains a 5?
6. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed, find the probability that the sum of the numbers that turn up is
   (*i*) 3
   (*ii*) 12.

   Hint. Sample space = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} where the first members of the ordered pairs represent the numbers on the coin and second members represent the numbers on the die.
7. If the odds in favour of an event are 2 : 1, what is the probability of its happening?
8. A bag contains 8 red discs and x blue discs. The odds against drawing a blue disc are 2 : 5. What is the value of x?
9. A committee of two persons is selected from two men and two women. What is the probability that the committee will have
   (*i*) no man
   (*ii*) one man
   (*iii*) two men?
10. When two balls are drawn simultaneously from a bag containing 5 white and 7 black balls, what is the probability that
    (*i*) both balls are white
    (*ii*) one ball is white and the other black?
11. If two letters are chosen at random from the English alphabet, find the probability that both are vowels?
12. If six persons sit in a row, what is the probability that A and B are sitting on adjacent seats?
13. A die has two faces with number 1, 1 face with number 3 and 3 faces with number 2. If the die is rolled once, find :
    (*i*) P(2)
    (*ii*) P(1 or 3)
    (*iii*) P(not 3).
14. Two dice are thrown.
    (*i*) What are the odds in favour of getting the sum 5?
    (*ii*) What are the odds against getting the sum 6?
15. Two cards are drawn simultaneously from a pack of 52 cards. Find the probability of
    (*i*) getting both red cards
    (*ii*) getting two red pictures.

16. In a lottery, a person chooses six different numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?

Hint. The order of numbers is not important.

17. Out of 9 outstanding students in a college, there are 4 boys and 5 girls. A team of four students is to be selected for a quiz programme. Find the probability that two are boys and two are girls.

18. A housewife buys a dozen eggs of which two turn out rotten. She chooses four eggs to scramble for breakfast. Find the chance that she chooses

(*i*) all good eggs (*ii*) three good and one bad

(*iii*) two good and two bad (*iv*) atleast one bad egg.

19. In a relay race, there are six teams A, B, C, D, E, F.

(*i*) What is the probability that A, B, C, D finish first, second, third and fourth respectively?

(*ii*) What is the probability that A, B, C and D are first four to finish (in any order)? Assume that all finishing orders are equally likely.

Answers

1.	(*i*) $\frac{1}{2}$	(*ii*) $\frac{2}{3}$	(*iii*) $\frac{1}{6}$	(*iv*) 0	(*v*) $\frac{5}{6}$
2.	(*i*) $\frac{1}{4}$	(*ii*) $\frac{3}{4}$	(*iii*) $\frac{1}{13}$	(*iv*) $\frac{12}{13}$	(*v*) $\frac{1}{52}$
	(*vi*) $\frac{1}{2}$	(*vii*) $\frac{1}{2}$	(*viii*) $\frac{4}{13}$	(*ix*) $\frac{11}{26}$	(*x*) $\frac{3}{26}$
3.	(*i*) $\frac{4}{9}$	(*ii*) $\frac{2}{9}$	(*iii*) $\frac{1}{3}$	(*iv*) $\frac{2}{3}$	(*v*) $\frac{2}{3}$
4.	(*i*) $\frac{6}{13}$	(*ii*) $\frac{7}{13}$	**5.** $\frac{3}{10}$	(*ii*) $\frac{2}{5}$	(*iii*) $\frac{1}{10}$
6.	(*i*) $\frac{1}{12}$	(*ii*) $\frac{1}{12}$	**7.** $\frac{2}{3}$	**8.** 20	
9.	(*i*) $\frac{1}{6}$	(*ii*) $\frac{2}{3}$	(*iii*) $\frac{1}{6}$	**10.** (*i*) $\frac{5}{33}$	(*ii*) $\frac{35}{66}$
11.	$\frac{2}{65}$	**12.** $\frac{1}{3}$	**13.** (*i*) $\frac{1}{2}$	(*ii*) $\frac{1}{2}$	(*iii*) $\frac{5}{6}$
14.	(*i*) 1 : 8	(*ii*) 31 : 5	**15.** (*i*) $\frac{25}{102}$	(*ii*) $\frac{5}{442}$	**16.** $\frac{1}{38760}$ **17.** $\frac{10}{21}$
18.	(*i*) $\frac{14}{33}$	(*ii*) $\frac{16}{33}$	(*iii*) $\frac{1}{11}$	(*iv*) $\frac{19}{33}$	**19.** (*i*) $\frac{1}{360}$ (*ii*) $\frac{1}{15}$