Example 4. *If* $f(x) = \log\left(\frac{1+x}{1-x}\right)$, *prove that* $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

Solution. Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$,

$$\therefore \quad f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right) = \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\frac{(1+x)^2}{(1-x)^2} = \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right) = 2f(x).$$

Example 5. *Find the domain of the function* $f(x) = \frac{1}{\log(3-x)}$.

Solution. For D_f, $f(x)$ must be a real number

$\Rightarrow \quad \frac{1}{\log(3-x)}$ must be a real number

$\Rightarrow \quad 3 - x > 0$ and $3 - x \neq 1$ $\qquad (\because \log 1 = 0)$

$\Rightarrow \quad 3 > x$ and $2 \neq x \Rightarrow x < 3$ and $x \neq 2$

$\Rightarrow \quad D_f = (-\infty, 2) \cup (2, 3)$.

EXERCISE 9.3

1. Solve the following equations for x:

   (i) $[2x - 3] = 5$ (ii) $[2x + 1] = -2$ (iii) $[2 - 3x] = 5$.

2. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then show that $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$.

3. Find the domain of the following functions:

   (i) $\log(4x - 3)$ (ii) $\frac{1}{\log(9-x^2)}$.

Answers

1. (i) $\left[4, \frac{9}{2}\right)$ (ii) $\left[-\frac{3}{2}, -1\right)$ (iii) $\left(-\frac{4}{3}, -1\right]$

3. (i) $\left(\frac{3}{4}, \infty\right)$ (ii) $(-3, 3) - \{-2\sqrt{2}, 2\sqrt{2}\}$

OPERATIONS ON REAL FUNCTIONS

The algebraic operations of addition, subtraction, multiplication and division etc. can be performed on two real valued functions *suitably* in the same manner as they are performed on two real numbers.

Let $f : X \to \mathbf{R}$ and $g : X \to \mathbf{R}$ be any two real functions where $X \subset \mathbf{R}$, then

(i) *the* ***sum*** *of f and g denoted by f + g, is the function defined by*

$(f + g)(x) = f(x) + g(x)$, *for all* $x \in X$.

(ii) *the* ***difference*** *of f and g denoted by f − g, is the function defined by*

$(f - g)(x) = f(x) - g(x)$, *for all* $x \in X$.

(iii) *the* **product** *of f and g denoted by fg, is the function defined by*

$(fg)(x) = f(x)\,g(x)$, *for all* $x \in X$.

(iv) *the* **quotient** *of f by g denoted by* $\frac{f}{g}$, *is the function defined by*

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ for all } x \in X_1 \text{ where } X_1 = \{x; x \in X, g(x) \neq 0\}.$$

Further, the function ff is usually denoted by f^2, the function f^2f or ff^2 by f^3 and so on and hence the **power function** f^n $(n \in N)$ is defined as $(f^n)(x) = (f(x))^n$, for all $x \in X$.

The function $\frac{1}{f}$ is called **reciprocal** of f and is defined as

$$\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)} \text{ with domain } X_2 = \{x; x \in X, f(x) \neq 0\}.$$

If c is any real number, then the function cf, called **scalar multiple** of f by c is defined as $(cf)(x) = cf(x)$, for all $x \in X$.

Remark

The functions f and g need not necessarily have the same domain. When their domains are different, then the operations $f+g$, $f-g$, fg and $\frac{f}{g}$ can be performed only on a domain which is common to both the functions. Take special care of $\frac{f}{g}$, this operation can be performed on a domain where $g(x) \neq 0$.

Composition of functions

It is an operation which arises naturally in case of functions and it has no analogue in real number system. In fact, we have been dealing with functions arising out of this composition but without taking a note of it. For example, the function $h(x) = \sqrt{4-x^2}$ is the **composite** of the functions $f(x) = 4 - x^2$ and $g(x) = \sqrt{x}$

i.e. $h(x) = g(f(x)) = g(4-x^2) = \sqrt{4-x^2}$

This leads to:

Let f, g be two real valued functions and let $D = \{x : x \in D_f, f(x) \in D_g\} \neq \phi$, then the composite of f and g, denoted by gof, is the function defined by $(gof)(x) = g(f(x))$ with domain D.

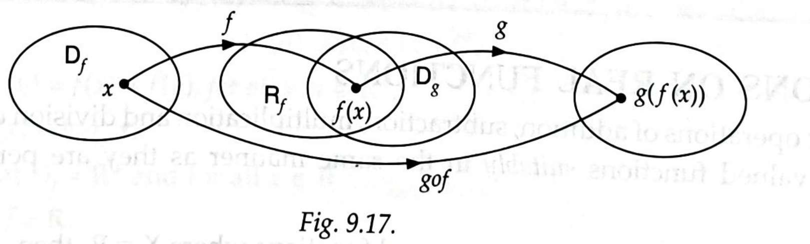


Fig. 9.17.

The *composite* of two functions is also called the ***resultant of two functions*** or ***function of a function***:

In particular, if $R_f \subset D_g$ then $D_{gof} = D_f$.

Remark

It may happen that *gof* may exist and *fog* may not exist. Moreover, even if both *gof* and *fog* exist, they may not be equal.

ILLUSTRATIVE EXAMPLES

Example 1. *Let $f(x) = x + 1$ and $g(x) = 2x - 3$ be two real functions. Find the following functions:*

(i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$ (v) $f^2 - 3g$.

Solution. Given $f(x) = x + 1$ and $g(x) = 2x - 3$.

We note that $D_f = \mathbf{R}$ and $D_g = \mathbf{R}$, so these functions have the same domain **R**.

(i) $(f+g)(x) = f(x) + g(x) = (x+1) + (2x-3) = 3x - 2$, for all $x \in \mathbf{R}$

(ii) $(f-g)(x) = f(x) - g(x) = (x+1) - (2x-3) = -x + 4$, for all $x \in \mathbf{R}$

(iii) $(fg)(x) = f(x)\,g(x) = (x+1)(2x-3) = 2x^2 - x - 3$, for all $x \in \mathbf{R}$

(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}, x \neq \frac{3}{2}, x \in \mathbf{R}$

(v) $(f^2 - 3g)(x) = (f^2)(x) - (3g)(x) = (f(x))^2 - 3g(x)$

$= (x+1)^2 - 3(2x-3) = x^2 + 2x + 1 - 6x + 9$

$= x^2 - 4x + 10$, for all $x \in \mathbf{R}$.

Example 2. *Let f, g be two functions defined by $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$, describe the following functions:*

(i) $f + g$ (ii) $g - f$ (iii) gf (iv) $\frac{f}{g}$

(v) $\frac{g}{f}$ (vi) $2f - \sqrt{5}\,g$ (vii) $f^2 + 7f$ (viii) $\frac{3}{g}$.

Solution. Given $f(x) = \sqrt{x+1}$, $g(x) = \sqrt{9-x^2}$

$\Rightarrow$ $D_f = [-1, \infty)$ and $D_g = [-3, 3]$.

Let $D = D_f \cap D_g = [-1, \infty) \cap [-3, 3] = [-1, 3] \neq \phi$, then

(i) $(f+g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{9-x^2}$ with domain $[-1, 3]$.

(ii) $(g-f)(x) = g(x) - f(x) = \sqrt{9-x^2} - \sqrt{x+1}$ with domain $[-1, 3]$.

(iii) $(gf)(x) = g(x)\,f(x) = \sqrt{9-x^2} - \sqrt{x+1} = \sqrt{(9-x^2)(x+1)}$ with domain $= [-1, 3]$.

(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{9-x^2}} = \sqrt{\frac{x+1}{9-x^2}}$ with domain D_1

where $D_1 = D$ except where $g(x) = 0 \Rightarrow D_1 = [-1, 3)$.

(v) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{9-x^2}}{\sqrt{x+1}} = \sqrt{\frac{9-x^2}{x+1}}$ with domain D_2

where $D_2 = D$ except where $f(x) = 0 \Rightarrow D_2 = (-1, 3]$.

(vi) $(2f - \sqrt{5}\,g)(x) = 2f(x) - \sqrt{5}\,g(x) = 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2}$ with domain $[-1, 3]$.

(vii) $(f^2 + 7f)(x) = (f(x))^2 + 7f(x) = x + 1 + 7\sqrt{x+1}$ with domain $[-1, \infty)$.

(viii) $\left(\frac{3}{g}\right)(x) = \frac{3}{g(x)} = \frac{3}{\sqrt{9-x^2}}$ with domain D_3

where $D_3 = D_g$ except where $g(x) = 0 \Rightarrow D_3 = (-3, 3)$.

Example 3. *Find the domain of the function f defined by* $f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$.

Solution. Let $f = g + h$, then $g(x) = \sqrt{4-x}$ and $h(x) = \frac{1}{\sqrt{x^2-1}}$.

For D_g, $g(x)$ must be a real number $\Rightarrow 4 - x \geq 0$

$\Rightarrow \quad 4 \geq x \Rightarrow x \leq 4 \Rightarrow D_g = (-\infty, 4]$.

For D_h, $h(x)$ must be a real number $\Rightarrow x^2 - 1 > 0$

$\Rightarrow \quad (x+1)(x-1) > 0 \Rightarrow x < -1$ or $x > 1$

$\Rightarrow \quad D_h = (-\infty, -1) \cup (1, \infty)$.

As $f = g + h$, so $D_f = D_g \cap D_h$

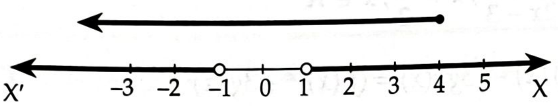


Fig. 9.18.

From fig. 9.19, it is clear that $D_g \cap D_h = (-\infty, -1) \cup (1, 4]$.

Hence, the domain of the function $f = (-\infty, -1) \cup (1, 4]$.

Example 4. *Find the domain of the function* $f(x) = \frac{1}{\log(1-x)} + \sqrt{x+3}$.

Solution. Let $f = g + h$, then $g(x) = \frac{1}{\log(1-x)}$ and $h(x) = \sqrt{x+3}$.

For D_g, $g(x)$ must be a real number

$\Rightarrow \quad \frac{1}{\log(1-x)}$ must be a real number

$\Rightarrow \quad 1 - x > 0$ and $1 - x \neq 1 \Rightarrow x < 1$ and $x \neq 0$

$\Rightarrow \quad D_g = (-\infty, 0) \cup (0, 1)$.

For D_h, $h(x)$ must be a real number

$\Rightarrow \quad \sqrt{x+3}$ must be a real number $\Rightarrow x + 3 \geq 0$

$\Rightarrow \quad x \geq -3$

$\Rightarrow \quad D_h = [-3, \infty)$

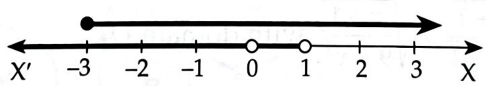


Fig. 9.19.

As $f = g + h$, so $D_f = D_g \cap D_h = [-3, 0) \cup (0, 1)$.

Example 5. *If* $f(x) = e^x$ *and* $g(x) = \log x$, *then find the following functions:*

(i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$

(v) $\frac{1}{f}$ (vi) f^2.

Solution. Given $f(x) = e^x$ and $g(x) = \log x$,

$D_f = \mathbf{R}$ and $D_g = \mathbf{R}^+$, where $\mathbf{R}^+$ is the set of positive reals.

Let $D = D_f \cap D_g = \mathbf{R} \cap \mathbf{R}^+ = \mathbf{R}^+ \neq \phi$, then

(i) $(f+g)(x) = f(x) + g(x) = e^x + \log x$, with domain $\mathbf{R}^+$.

(ii) $(f-g)(x) = f(x) - g(x) = e^x - \log x$, with domain $\mathbf{R}^+$.

(iii) $(fg)(x) = f(x)\, g(x) = e^x \log x$, with domain $\mathbf{R}^+$.

(iv) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{e^x}{\log x}$ with domain D_1

where $D_1 = \mathbf{R}^+$ except $g(x) = 0$ *i.e.* except $\log x = 0$ *i.e.* except $x = 1$

$\Rightarrow\ D_1 = \mathbf{R}^+ - \{1\}$.

(v) $\left(\dfrac{1}{f}\right)(x) = \dfrac{1}{f(x)} = \dfrac{1}{e^x}$, with domain $\mathbf{R}$ except $f(x) = 0$ *i.e.* with domain $\mathbf{R}$.

(vi) $(f^2)(x) = (f(x))^2 = (e^x)^2 = e^{2x}$, with domain $D_f = \mathbf{R}$.

Example 6. *If* $f(x) = x - \dfrac{1}{x}$, *prove that* $(f(x))^3 = f(x^3) + 3f\left(\dfrac{1}{x}\right)$.

Solution. Given $f(x) = x - \dfrac{1}{x}$, $D_f = \mathbf{R} - \{0\}$

$$\Rightarrow\quad f(x^3) = x^3 - \frac{1}{x^3} \text{ and } f\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{\frac{1}{x}} = \frac{1}{x} - x \qquad \ldots(i)$$

$$\therefore\quad (f(x))^3 = \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x.\frac{1}{x}\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} + 3\left(\frac{1}{x} - x\right) = f(x^3) + 3f\left(\frac{1}{x}\right) \qquad \text{(using } (i))$$

Example 7. *If* $f(x) = x^2 - 2$, $g(x) = 2x + 3$, *then describe the following functions:*

(i) fog (ii) gof (iii) fof (iv) gof

Solution. Given $f(x) = x^2 - 2 \Rightarrow D_f = \mathbf{R}$, $R_f = [-2, \infty)$

and $g(x) = 2x + 3 \Rightarrow D_g = \mathbf{R}$, $R_g = \mathbf{R}$

(i) $fog\,(x) = f(g(x)) = f(2x+3)$
$= (2x+3)^2 - 2 = 4x^2 + 12x + 7$

(ii) $gof\,(x) = g(f(x)) = g(x^2 - 2)$
$= 2(x^2 - 2) + 3 = 2x^2 - 1$

(iii) $fof\,(x) = f(f(x)) = f(x^2 - 2)$
$= (x^2 - 2)^2 - 2 = x^4 - 4x^2 + 2$

(iv) $gog\,(x) = g(g(x)) = g(2x+3)$
$= 2(2x+3) + 3 = 4x + 9$

Note that as $D_f = \mathrm{R}$ and $D_g = \mathbf{R}$, all the above four composite functions exist and domain of each $= \mathbf{R}$.

EXERCISE 9.4

1. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, then find the values of

   (i) $f(3) + g(-5)$ (ii) $f(-2) + g(-1)$ (iii) $f\left(\frac{1}{2}\right) \times g(14)$ (iv) $f(t) - f(-2)$

2. If $f(x) = e^x$ and $g(x) = \log x$, then find:

   (i) $(f - g)(1)$ (ii) $(fg)(1)$ (iii) $\left(\frac{f}{g}\right)(3)$.

3. If $f(x) = x^3 + 1$ and $g(x) = x + 1$ be two real functions, then find the following functions:

   (i) $f + g$ (ii) $g - f$ (iii) fg (iv) $\frac{f}{g}$ (v) $2g^2 - 3f$.

4. If $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{x^2 - 1}$ be two real valued functions, then find the following functions:

   (i) $f + g$ (ii) $g - f$ (iii) fg (iv) $3f - 2g$

   (v) $\frac{f}{g}$ (vi) $2f^2 + \sqrt{3}g$ (vii) $\frac{1}{f}$ (viii) $\frac{g}{f}$.

5. If $f(x) = x^3 - \frac{1}{x^3}$, prove that $f(x) + f\left(\frac{1}{x}\right) = 0$.

6. If $f(x) = x + \frac{1}{x}$, prove that $(f(x))^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$.

7. If $f(x) = 2x + 5$ and $g(x) = x^2 + 1$, describe the functions:

   (i) fog (ii) gof (iii) fof (iv) gog (v) f^2

   Also show that $fof \neq f^2$.

8. If $f(x) = \frac{3x-2}{2x-3}$, prove that $(fof)(x) = x, x \neq \frac{3}{2}$.

Answers

1. (i) 6 (ii) 13 (iii) $\frac{1363}{4}$ (iv) $t^2 - 4$

2. (i) e (ii) 0 (iii) $\frac{e^3}{\log 3}$

3. (i) $x^3 + x + 2, x \in \mathbf{R}$ (ii) $x - x^3, x \in \mathbf{R}$ (iii) $x^4 + x^3 + x + 1, x \in \mathbf{R}$

   (iv) $\frac{x^3 + 1}{x + 1}, x \in \mathbf{R}, x \neq -1$ (v) $2x^2 + 4x - 3x^3 - 1, x \in \mathbf{R}$

4. (i) $\sqrt{x-2} + \sqrt{x^2 - 1}$ (ii) $\sqrt{x^2 - 1} - \sqrt{x - 2}$ (iii) $\sqrt{(x-2)(x^2-1)}$

   (iv) $3\sqrt{x-2} - 2\sqrt{x^2-1}$ (v) $\sqrt{\frac{x-2}{x^2-1}}$ (vi) $2(x-2) + \sqrt{3(x^2-1)}$

   (vii) $\frac{1}{\sqrt{x-2}}$ (viii) $\sqrt{\frac{x^2-1}{x-2}}$

   Domain in case (i) to (vi) is $[2, \infty)$ and in cases (vii), (viii) is $(2, \infty)$

7. (i) $2x^2 + 7$ (ii) $4x^2 + 20x + 26$ (iii) $4x + 15$ (iv) $x^4 + 2x^2 + 2$

   (v) $4x^2 + 20x + 25$

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options in questions (1 to 14):

1. Which of the following relations is a function?
   (*a*) $R = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$ (*b*) $R = \{(1, 2), (2, 4), (2, 6), (3, 5)\}$
   (*c*) $R = \{(2, 1), (4, 3), (6, 5), (8, 7), (10, 9)\}$ (*d*) $R = \{(0, 1), (1, 3), (2, 4), (3, 1), (3, 5)\}$

2. Let A and B be two finite sets, then the number of functions from A to B is
   (*a*) $n(A).n(B)$ (*b*) $2^{n(A).n(B)}$ (*c*) $\{n(A)\}^{n(B)}$ (*d*) $\{n(B)\}^{n(A)}$

3. Let A be a finite set containing 3 elements, then the number of functions from A to A is
   (*a*) 512 (*b*) 511 (*c*) 27 (*d*) 26

4. The domain of the function *f* defined by $f(x) = \sqrt{a^2 - x^2}$ $(a > 0)$ is
   (*a*) $(-a, a)$ (*b*) $[-a, a]$ (*c*) $[0, a]$ (*d*) $(-a, 0]$

5. The domain of the function *f* defined by $f(x) = \sqrt{x^2 - 9}$ is
   (*a*) $[-3, 3]$ (*b*) $(-3, 3)$ (*c*) $(-\infty, -3] \cup [3, \infty)$ (*d*) $[0, 3]$

6. The domain of the function *f* defined by $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$ is
   (*a*) $\mathbf{R} - [3, -2]$ (*b*) $\mathbf{R} - \{-3, 2\}$ (*c*) $\mathbf{R} - \{3, -2\}$ (*d*) $\mathbf{R} - (-3, 2)$

7. The domain and range of the real function *f* defined by $f(x) = \sqrt{x - 1}$ are
   (*a*) Domain = $(1, \infty)$, Range = $(0, \infty)$ (*b*) Domain = $[1, \infty)$, Range = $(0, \infty)$
   (*c*) Domain = $[1, \infty)$, Range = $[0, \infty)$ (*d*) Domain = $(1, \infty)$, Range = $[0, \infty)$

8. The domain and range of the real function *f* defined by $f(x) = \dfrac{x - 2}{2 - x}$ are
   (*a*) Domain = $\mathbf{R} - \{2\}$, Range = $\{-1\}$ (*b*) Domain = $\mathbf{R} - \{-2\}$, Range = $\{-1\}$
   (*c*) Domain = $\mathbf{R} - \{-2\}$, Range = $\{1\}$ (*d*) Domain = $\mathbf{R} - \{2\}$, Range = $\{1\}$

9. The domain and range of the real function *f* defined by $\dfrac{x}{|x|}$ are
   (*a*) Domain = $\mathbf{R}$, Range = $\{-1, 1\}$ (*b*) Domain = $\mathbf{R} - \{0\}$, Range = $\{-1, 0, 1\}$
   (*c*) Domain = $\mathbf{R} - \{0\}$, Range = $\{-1, 1\}$ (*d*) Domain = $\mathbf{R}$, Range = $\{-1, 0, 1\}$

10. The domain and range of the function *f* given by $f(x) = 2 - |x - 5|$ are
    (*a*) Domain = $\mathbf{R}^+$, Range = $(-\infty, 1]$ (*b*) Domain = $\mathbf{R}$, Range = $(-\infty, 2]$
    (*c*) Domain = $\mathbf{R}^+$, Range = $(-\infty, 2]$ (*d*) Domain = $\mathbf{R}$, Range = $(-\infty, 2)$

11. The domain of the function *f* defined by $f(x) = \sqrt{a - x} + \dfrac{1}{\sqrt{x^2 - a^2}}$ is
    (*a*) $(-\infty, a]$ (*b*) $(-\infty, -a]$ (*c*) $(-\infty, -a)$ (*d*) (a, ∞)

12. The domain of the function *f* defined by $f(x) = \log_e(5 - 6x)$ is
    (*a*) $\left(-\infty, \frac{5}{6}\right)$ (*b*) $\left(\frac{5}{6}, \infty\right)$ (*c*) $\left(-\infty, \frac{5}{6}\right]$ (*d*) $\left[\frac{5}{6}, \infty\right)$

13. If $f(x) = px + q$, where *p* and *q* are integers $f(-1) = 1$ and $f(2) = 13$, then *p* and *q* are
    (*a*) $p = 4, q = 5$ (*b*) $p = -4, q = 5$ (*c*) $p = -4, q = -5$ (*d*) $p = 4, q = -5$

14. The domain for which the functions defined by $f(x) = 6x^2 + 1$ and $g(x) = 11 - 7x$ are equal is
    (*a*) $\left\{-1, \frac{2}{3}\right\}$ (*b*) $\left\{3, \frac{5}{6}\right\}$ (*c*) $\left\{-2, \frac{5}{6}\right\}$ (*d*) $\left\{2, \frac{2}{3}\right\}$

Answers

1. (c) 2. (d) 3. (c) 4. (b) 5. (c) 6. (c) 7. (c)
8. (a) 9. (c) 10. (b) 11. (c) 12. (a) 13. (a) 14. (c)

FILL IN THE BLANKS

Fill in the blanks in questions (1 to 14):

1. If $n(A) = 4$ and $n(B) = 3$, then total number of functions from A to B is ....
2. The value of the function $f(x) = |x-2| + |2+x|$ for $-2 \le x \le 2$ is ....
3. The domain of the function $f(x) = \sqrt{x^2 - 4x + 3}$ is ....
4. The domain of the function $f(x) = \dfrac{1}{\log_e x}$ is ....
5. The range of the function $f(x) = |3x+2| - 4$ is ....
6. The range of the function $f(x) = \dfrac{x}{x+1}, x \ge 0$ is ....
7. If $(p, 11)$ and $(2, q)$ are ordered pairs which belong to the function $f : x \to (x^2 + 2)$, where $x \in \mathbf{R}^+$, then the values of p and q are ....
8. Let f and g be two functions given by $f = \{(2, 4), (5, 6), (8, -1)\ (10, -3)\}$ and $g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, -5)\}$, then the domain of $f + g$ is ....
9. Let f and g be two real functions given by $f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$, $g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$, then the domain of $f.\,g$ is ....
10. Let $f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$, $g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$ be two real functions, then $\dfrac{f}{g}$ is equal to ....
11. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two real functions, then the domain of $\dfrac{f}{g}$ is equal to ....
12. If f and g are two real functions defined by $f(x) = 5x + 4$ and $g(x) = 3x^3 - 4x + 7$, then the value of $f(3) + g(-2) =$ ....
13. If $f(x) = \log\left(\dfrac{a-x}{a+x}\right)$, then $f(x) + f(-x) =$ ....
14. If $f(x) = a^x + a^{-x}$, then $(f(x))^2 - f(2x) =$ ....

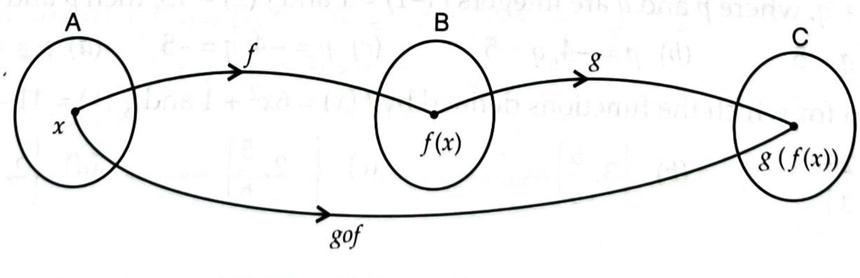
Answers

1. 81 2. 4 3. $(-\infty, 1] \cup [3, \infty)$ 4. $(0, 1) \cup (1, \infty)$
5. $[-4, \infty)$ 6. $[0, 1)$ 7. $p = 3, q = 6$ 8. $\{2, 8, 10\}$ 9. $\{2, 3, 4, 5\}$
10. $\left[\left(2, \frac{4}{5}\right), \left(8, -\frac{1}{4}\right), \left(10, -\frac{3}{13}\right)\right]$ 11. $(0, \infty)$ 12. 10 13. 0 14. 2

CASE-STUDY BASED QUESTIONS (Unsolved)

1. Let $f : A \to B$ and $g : B \to C$ be two real functions, then composite of f and g denoted by gof is the function $gof : A \to C$ defined by

$$(gof)(x) = g(f(x)), \forall x \in A$$



For gof to exist, Range of $f \subset$ Domain of g

Also, $D_{gof} = D_f$

The functions f and g are given by

$f = \{(0, 4), (2, 5), (3, 7)\}$ and $g = \{(4, 2), (5, 0), (7, 2)\}$.

Based on the above information, answer the following questions:

(i) Domain of f is

(a) $\{4, 5, 7\}$ (b) $\{4, 0, 2\}$ (c) $\{0, 2, 3\}$ (d) $\{0, 2\}$

(ii) Domain of g is

(a) $\{4, 5, 7\}$ (b) $\{4, 0, 2\}$ (c) $\{0, 2, 3\}$ (d) $\{0, 2\}$

(iii) Range of f and g are respectively

(a) $\{0, 2, 3\}$ and $\{0, 2\}$ (b) $\{4, 5, 7\}$ and $\{0, 2\}$

(c) $\{0, 2, 3\}$ and $\{4, 5, 7\}$ (d) $\{0, 2\}$ and $\{4, 5, 7\}$

(iv) gof as a set of ordered pairs is

(a) $\{(0, 2), (2, 0), (3, 2)\}$ (b) $\{(0, 2), (2, 2), (7, 5)\}$

(c) $\{(4, 4), (5, 5), (7, 7)]$ (d) $\{(4, 5), (5, 4), (7, 5)\}$

(v) fog as a set of ordered pairs is

(a) $\{(0, 2), (2, 0), (3, 2)\}$ (b) $\{(0, 2), (2, 2), (7, 5)\}$

(c) $\{(4, 4), (5, 5), (7, 7)\}$ (d) $\{(4, 5), (5, 4), (7, 5)\}$

2. Let f and g be two real functions defined by $f(x) = \sqrt{x - 1}$ and $g(x) = 3 - 2x$.

Based on the above information, answer the following questions:

(i) Domain of f is

(a) $(1, \infty)$ (b) $[1, \infty)$ (c) $(-\infty, 1)$ (d) $(-\infty, 1]$

(ii) Domain of $\frac{1}{g}$ is

(a) $\mathbf{R} - \left\{\frac{3}{2}\right\}$ (b) $\mathbf{R}$ (c) $\mathbf{R} - \left\{\frac{2}{3}\right\}$ (d) $\mathbf{R} - \left\{-\frac{3}{2}\right\}$

(iii) Domain of $f + g$ is

(a) $\mathbf{R}$ (b) $\mathbf{R} - (1, \infty)$ (c) $[1, \infty)$ (d) $\mathbf{R} - \left\{\frac{2}{3}\right\}$

(iv) Domain of $\frac{g}{f}$ is

(a) $(1, \infty)$ (b) $[1, \infty)$ (c) $\mathbf{R} - \left\{\frac{3}{2}\right\}$ (d) $\mathbf{R}$

(v) Domain of fog is

(a) $(1, \infty)$ (b) $[1, \infty)$ (c) $(-\infty, 1)$ (d) $(-\infty, 1]$

Answers

1. (i) (c) (ii) (a) (iii) (b) (iv) (a) (v) (d)
2. (i) (b) (ii) (a) (iii) (c) (iv) (a) (v) (d)