12 Perpetuity, Sinking Funds and EMI

PERPETUITY AND SINKING FUNDS

In class XI, you have learnt about present value and future value of a regular annuity. In this chapter, you will learn about two more type of annuities, perpetuity and sinking funds.

PERPETUITY

A perpetuity is an annuity in which the periodic payments begin on a fixed date and continue forever. It is also called **perpetual annuity.**

For example, a person purchases a house (or flat) and rents it out. The person is entitled to get rent as long as the property continues to exist (assuming that property continues to exist forever). Another example of perpetuity can be from stock market. Suppose a person invested in shares of a company, then he/she is entitled to get dividend from the company assuming that company will continue to exist forever.

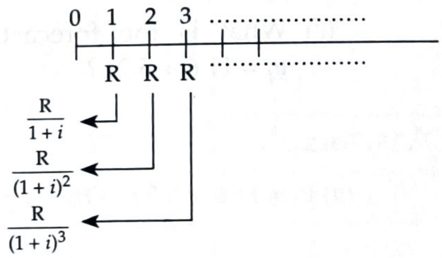
As the term of the perpetuity is infinite, so the amount or future value of perpetuity is undefined. There are two types of perpetuity one in which the payment is made at the end of each payment period and other in which the payment is made at the beginning of each payment period.

Present Value of a Perpetuity

Let us consider an annuity whose periodic payment is ₹R for infinite periods, interest being $r\%$ per period or $\frac{r}{100} = i$ per period per rupee.

Type I: When periodic payment is made at the end of each payment period.

To understand the calculations of present values of all the payments see the adjoining diagram.



The present value of first payment $P_1 = \frac{R}{1+i}$

The present value of second payment $P_2 = \frac{R}{(1+i)^2}$

The present value of third payment $P_3 = \frac{R}{(1+i)^3}$

... ...

... ...

On adding present values of all the payments, we get the present value of the annuity which is denoted by P

$$P = P_1 + P_2 + P_3 + \ldots$$

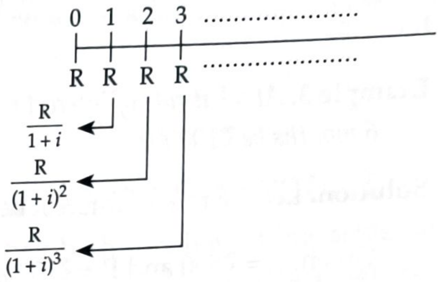
$$\Rightarrow \quad P = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots$$

It is an infinite geometric series with $a = \dfrac{R}{1+i}$ and $r = \dfrac{1}{1+i}$.

$$\therefore \quad P = \frac{\frac{R}{1+i}}{1 - \frac{1}{1+i}} = \frac{\frac{R}{1+i}}{\frac{1+i-1}{1+i}}$$

$$\Rightarrow \quad P = \frac{R}{i}$$

Type II: When periodic payment is made at the beginning of each payment period.



To understand the calculations of present values of all the payments see the adjoining diagram.

Here, the present value of first payment $P_1 = R$

The present value of second payment $P_2 = \dfrac{R}{1+i}$

The present value of third payment $P_3 = \dfrac{R}{(1+i)^2}$

The present value of fourth payment $P_4 = \dfrac{R}{(1+i)^3}$

... ...

... ...

So, $\quad P = P_1 + P_2 + P_3 + P_4 + \ldots$

$$\Rightarrow \quad P = R + \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots$$

It is an infinite geometric series with $a = R$ and $r = \dfrac{1}{1+i}$.

$$\therefore \quad P = \frac{R}{1 - \frac{1}{1+i}} = \frac{R(1+i)}{1+i-1} \Rightarrow P = \frac{R(1+i)}{i}$$

$$\Rightarrow \quad P = R + \frac{R}{i}.$$

ILLUSTRATIVE EXAMPLES

Example 1. *Find the present value of a sequence of payments of ₹1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.*

Solution. The given annuity is a perpetuity of first type in which R = ₹1000 and $r = \frac{8}{2}\% = 4\%$ per half year.

So, $\quad i = \dfrac{4}{100} = 0.04$

Present value, $P = \dfrac{R}{i} = \dfrac{1000}{0.04} = 25000$

Hence, the present value is ₹25000.

Example 2. *What sum of money is needed to invest now, so as to get ₹5000 at the beginning of every month forever, if the money is worth 6% per annum compounded monthly?*

Solution. Given R = ₹5000, $r = \frac{6}{12}\% = 0.5\%$ per month.

So, $i = \frac{0.5}{100} = 0.005$

The given annuity is a perpetuity of second type.

So, $P = R + \frac{R}{i} = 5000 + \frac{5000}{0.005}$

$= 5000 + 1000000 = 1005000.$

Hence, ₹1005000 are needed to invest now to get ₹5000 at the beginning of every month forever.

Example 3. *At what rate of interest will the present value of a perpetuity of ₹500 payable at the end of every 6 months be ₹10000?*

Solution. Let the rate of interest be r% per annum, then $i = \frac{r}{200}$.

Given, R = ₹500 and P = ₹10000.

Using, $P = \frac{R}{i} \Rightarrow i = \frac{R}{P}$.

$\Rightarrow \quad \frac{r}{200} = \frac{500}{10000}$

$\Rightarrow \quad r = 10$

Hence, rate of interest = 10% per annum.

EXERCISE 12.1

1. Find the present value of a perpetuity of ₹900 payable at the end of each year, if money is worth 5% per annum.
2. Find the present value of a perpetuity of ₹500 payable at the end of each quarter, if money is worth 8% per annum.
3. Find the present value of a perpetuity of ₹300 payable at the beginning of every 6 months, if money is worth 6% per annum.
4. Find the present value of a perpetuity of ₹18000 payable at the end of 6 months, if the money is worth 8% p.a. compounded semi-annually. *(CBSE 2022 Term II)*
5. What amount is received at the end of every 6 months forever, if ₹72000 kept in a bank earns 8% per annum compounded half yearly?
6. At what rate of interest will the present value of a perpetuity of ₹300 payable at the end of each quarter be ₹24000?
7. A man wants to deposit a lump sum amount so that an annual scholarship of ₹3000 is paid. Rate of interest is 5% per annum. Calculate the lump sum amount required, if the scholarship is to start at the end of this year and continue forever.
8. What sum of money invested now could establish a scholarship of ₹5000 which is to be awarded at the end of every year forever, if money is worth 8% per annum.
9. ₹2,50,000 cash is equivalent to a perpetuity of ₹7500 payable at the end of each quarter. What is the rate of interest convertible quarterly?

Answers

1. ₹18000 2. ₹25000 3. ₹10300 4. ₹450000 5. ₹2880 6. 5%
7. ₹60000 8. ₹62500 9. 12%

SINKING FUND

A sinking fund is a fund created to accumulate money over the years to discharge a future obligation like replacing an old machine by a new machine, renovation of house, expansion of business etc.

The problems of sinking fund can be solved by using the formulas of regular annuity studied in class XI.

Let us consider an annuity whose periodic payment is ₹R payable at the end of each payment period for n periods, interest being r% per period or $i = \frac{r}{100}$, then the amount of obligation which can be discharged is $A = R\left[\frac{(1+i)^n - 1}{i}\right]$ or $A = R\,S_{\overline{n}|i}$.

DIFFERENCE BETWEEN SINKING FUND AND SAVINGS ACCOUNT

Sinking fund and savings account are both financial tools. They both involve setting aside an amount of money for the future. The differences between sinking fund and saving account are given below:

Sinking Fund	Savings Account
1. It is a fixed term account.	1. It is a long term account which can be closed any time.
2. It is set-up for a particular upcoming expense.	2. It does not have any specific purpose.
3. In sinking fund a fixed amount at regular intervals is deposited.	3. In savings account any amount, any time can be deposited.
4. It can be used only for the purpose it was created.	4. It can be used in any emergency.

ILLUSTRATIVE EXAMPLES

Example 1. *A firm anticipates an expenditure of ₹500000 for plant modernization at end of 10 years from now. How much should the company deposit at the end of each year into a sinking fund earning interest 5% per annum?*

Solution. Given, A = ₹500000, $r = 5\% \Rightarrow i = 0.05$ and $n = 10$, R = ?

Using formula, $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$$500000 = R\left[\frac{(1.05)^{10} - 1}{0.05}\right]$$

$$\Rightarrow \quad R = \frac{500000 \times 0.05}{(1.05)^{10} - 1}$$

[Let $x = (1.05)^{10}$

Taking logarithm on both sides, we get

$\log x = 10 \log 1.05 = 10 \times 0.0212 = 0.2120$

$\Rightarrow \quad x = \text{antilog } 0.2120$

$\Rightarrow \quad x = 1.629$]

$$\Rightarrow \quad R = \frac{25000}{1.629 - 1} = \frac{25000}{0.629}$$

$$\Rightarrow \quad R = 39745.63$$

Hence, the company should deposit ₹39745.63 every year into the sinking fund.

Alternatively. Using FVAF tables (annuity tables)

$$A = RS_{\overline{n}/i} \Rightarrow R = \frac{A}{S_{\overline{n}/i}} = \frac{500000}{S_{10/0.05}}$$

$$\Rightarrow \quad R = \frac{500000}{12.5779} = 39752.26$$

Note

Some difference in answers must be seen while using two different methods *i.e.* using log tables or using annuity tables.

Example 2. *A machine costing ₹200000 has effective life of 7 years and its scrap value is ₹30000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost ₹300000 after 7 years.* *(CBSE 2022 Term II)*

Solution. Cost of new machine = ₹300000

Scrap value of old machine = ₹30000

Hence, the money required for new machine after 7 years

= ₹300000 – ₹30000 = ₹270000

So, we have A = ₹270000, $i = \frac{5}{100} = 0.05$, $n = 7$

Using formula, $A = R\left[\frac{(1+i)^n - 1}{i}\right]$, we get

$$270000 = R\left[\frac{(1.05)^7 - 1}{0.05}\right]$$

$$\Rightarrow \quad R = \frac{270000 \times 0.05}{(1.05)^7 - 1}$$

$$\Rightarrow \quad R = \frac{13500}{1.407 - 1} = \frac{13500}{0.407}$$

$$\Rightarrow \quad R = 33169.53$$

[Let $x = (1.05)^7$

$\Rightarrow$ $\log x = 7 \log 1.05 = 7 \times 0.0212 = 0.1484$

$\Rightarrow$ $x = \text{antilog } 0.1484$

$\Rightarrow$ $x = 1.407$]

Hence, the company should deposit ₹33169.53 at the end of each year for 7 years.

Alternatively. Using annuity tables

$$A = RS_{\overline{n}/i} \Rightarrow R = \frac{A}{S_{\overline{n}/i}}$$

$$\Rightarrow \quad R = \frac{270000}{S_{7/0.05}} = \frac{270000}{8.1420} = 33161.39$$

Example 3. *Mr X plans to save amount for higher studies of his son, required after 10 years. He expects the cost of these studies to be ₹100000. How much should he save at the beginning of each year to accumulate this amount at the end of 10 years, if the interest rate is 12% compounded annually?*

Solution. Given A = ₹100000, $r = 12\% \Rightarrow i = 0.12$ and $n = 10$, R = ?

This is a case of annuity due, since payments are being made at the beginning of each year. So, using the formula of annuity due,

$$A = R(1+i)\left[\frac{(1+i)^n - 1}{i}\right]$$

or $\quad A = R(S_{\overline{n+1}/i} - 1)$ (For use of annuity table)

$$100000 = R\left[\frac{(1.12)^{11} - 1.12}{0.12}\right]$$

$$\Rightarrow \quad R = \frac{100000 \times 0.12}{[(1.12)^{11} - 1.12]}$$

[Let $x = (1.12)^{11}$

Taking logarithm on both sides, we get

$$\log x = 11 \log 1.12 = 11 \times 0.0492$$

$$\Rightarrow \quad \log x = 0.5412 \Rightarrow x = \text{antilog } 0.5412 \Rightarrow x = 3.477]$$

$$\therefore \quad R = \frac{12000}{3.477 - 1.12} = \frac{12000}{2.357}$$

$$\Rightarrow \quad R = 5091.22$$

Hence, Mr X should save ₹5091.22 at the beginning of each year to accumulate ₹100000.

EXERCISE 12.2

1. A company establishes a sinking fund to provide for payment of ₹250000 debt, maturing in 4 years. Contribution to the fund are to be made at the end of each year. Find the amount of each annual deposit if the interest is 18% per annum.
2. A person has set up a sinking fund so that he can accumulate ₹100000 in 10 years for his children's higher education. How much amount should he deposit every six months if interest is 5% per annum compounded semi-annually?
3. A firm anticipates a capital expenditure of ₹50000 for a new equipment in 5 years. How much should be deposited quarterly in a sinking fund carrying 12% per annum compounded quarterly to provide for the purchase?
4. A machine costs a company ₹525000 and its effective life is estimated to be 20 years. A sinking fund is created for replacing the machine at the end of its lifetime when its scrap realizes a sum of ₹25000 only. Calculate what amount should be provided every year out of profits for the sinking fund if it accumulates an interest of 5% per annum.
5. A machine being used by a company is estimated to have a life of 15 years. At that time a new machine would cost ₹75000 and the scrap of the old machine would yield ₹9600 only. A sinking fund is created for replacing the machine at the end of its life. What sum should be retained by the company at the end of every year to accumulate at 6% per annum?
6. A company sets aside a sum of ₹1000 at the end of each year in a sinking fund so that at the end of 10 years, it would amount to a balance sufficient to replace the machinery. Assuming that the cost of machinery remains constant at the end of 10 years and that money earns 5% per annum compound interest, find the cost of the machinery.
7. 10 years ago, Mr Mehra set-up a sinking fund to save for his daughter's higher studies. At the end of 10 years, he received an amount of ₹10,21,760. What amount did he put in the sinking fund at the end of every 6 months for the tenure, which paid him 5% p.a. compounded semi-annually? [Use $(1.025)^{20} = 1.6386$]

Answers

1. ₹47923.32 2. ₹3924.64 3. ₹1868 4. ₹15105.74
5. ₹2810.89 6. ₹12580 7. ₹40,000

EQUATED MONTHLY INSTALLMENT (EMI)

Nowadays loans have become an integral part of our life. Most of us take loans such as car loan, home loan, education loan, personal loan etc. from different financial institutions. However, when we talk about loans, we come across a term EMI which means Equated Monthly Installment. EMI is the monthly amount which we pay towards the repayment of the loan. EMI are used to pay off both interest on the loan and principal amount. Initially interest on the loan is the major part of EMI but as we progress towards the loan tenure the interest part reduces and principal part increases.

An equated monthly installment (EMI) is a fixed payment made by a borrower to a lender at a specified date every month to clear off the loan.

The EMI depends on the following factors:

(*i*) Principal borrowed

(*ii*) Rate of interest

(*iii*) Tenure of the loan.

Calculation of EMI

There are two methods of calculating EMI:

(*i*) Flat rate method

(*ii*) Reducing balance method.

Flat rate method

In flat rate method, the amount of interest paid is fixed. The amount of interest is calculated on the principal loan amount borrowed for its tenure at a constant rate of interest. In this method, the interest remains constant for every EMI and does not reduce as the outstanding principal amount decreases with every EMI.

In flat rate method, the EMI is calculated by adding the principal loan amount borrowed and the interest on the principal loan amount for loan tenure at a constant rate and the result is divided by the number of payments.

$$\text{EMI} = \frac{\text{Principal + Interest}}{\text{Number of payments}}$$

$$\text{EMI} = \frac{\text{P} + \text{P}ni}{n}$$

where P = Principal borrowed

i = interest rate per rupee per month

n = number of payments.

ILLUSTRATIVE EXAMPLES

Example 1. *Sanjay takes a personal loan of ₹500000 at the rate of 12% per annum for 3 years. Calculate his EMI by using flat rate method.*

Solution. Given P = ₹500000, $i = \frac{12}{12 \times 100} = 0.01$

and $n = 12 \times 3 = 36$.

$$\text{So, EMI} = \frac{\text{P} + \text{P}ni}{n} = \frac{500000 + 500000 \times 36 \times 0.01}{36}$$

$$= \frac{500000 + 180000}{36} = \frac{680000}{36}$$

$$= 18888.89$$

Hence, EMI = ₹18888.89

REDUCING BALANCE METHOD

Unlike flat rate method, interest does not remain constant for every EMI. As we pay off our loan, the outstanding principal amount decreases with every EMI and the interest for every subsequent EMI is calculated on the outstanding principal amount.

The EMI is calculated by reducing balance method using the following formula:

$$\text{EMI} = \frac{P \times i \times (1+i)^n}{(1+i)^n - 1} \text{ or EMI} = P\,\frac{i}{1-(i+i)^{-n}} \Rightarrow \text{EMI} = \frac{P}{a_{\overline{n}/i}}$$

where P = Principal borrowed

i = interest rate per rupee per month

n = number of payments.

Example 2. *Mr Sharma borrowed ₹1000000 from a bank to purchase a house and decided to repay the loan by equal monthly installments in 10 years. If bank charges interest at 9% p.a. compounded monthly, calculate the EMI. (Given $(1.0075)^{120} = 2.4514$)*

Solution. Given, $P = ₹1000000$, $i = \frac{9}{12 \times 100} = 0.0075$

and $n = 12 \times 10 = 120$

$$\text{So, EMI} = \frac{P \times i \times (1+i)^n}{(1+i)^n - 1} = \frac{1000000 \times 0.0075 \times (1.0075)^{120}}{(1.0075)^{120} - 1}$$

$$= \frac{7500 \times 2.4514}{2.4514 - 1} = \frac{7500 \times 2.4514}{1.4514} = 12667.42$$

Hence, EMI = ₹12667.42

Example 3. *A person wishes to purchase a house for ₹4500000 with a down payment of ₹500000 and balance in EMI for 25 years. If bank charges 6% per annum compounded monthly, calculate the EMI. (Given $(1.005)^{300} = 4.4650$)*

Solution. Cost of house = ₹4500000

Down payment = ₹500000

∴ Balance = ₹4000000

So, $P = ₹4000000$, $i = \frac{6}{12 \times 100} = 0.005$

and $n = 25 \times 12 = 300$

$$\therefore \quad \text{EMI} = \frac{4000000 \times 0.005 \times (1.005)^{300}}{(1.005)^{300} - 1}$$

$$= \frac{20000 \times 4.4650}{4.4650 - 1} = \frac{20000 \times 4.4650}{3.4650} = 25772$$

Hence, EMI = ₹25772.

Example 4. *Madhu exchanged her old car valued at ₹ 1,50,000 with a new one priced at ₹ 6,50,000. She paid ₹ x as down payment and the balance in 20 monthly equal instalments of ₹ 21,000 each. The rate of interest offered to her is 9% p.a. Find the value of x.*

[Given that: $(1.0075)^{-20} = 0.86118985$] *(CBSE 2022 Term II)*

Solution. Given price of old car = ₹ 1,50,000

and price of new car = ₹ 6,50,000.

So, Madhu has to pay ₹ (650000 – 150000) *i.e.* ₹ 5,00,000

Given down payment = ₹ x and EMI = ₹ 21000

$$n = 20 \text{ months}, \ i = \frac{9}{12 \times 100} = 0.0075$$

So, $P = ₹(500000 - x)$

Now, $\text{EMI} = P \dfrac{i}{1-(1+i)^{-n}}$

$$\Rightarrow \quad 2100 = (500000 - x) \cdot \frac{0.0075}{1-(1,0075)^{-20}}$$

$$\Rightarrow \quad 500000 - x = \frac{21000 \times (1 - 0.86118985)}{0.0075}$$

$$\Rightarrow \quad 500000 - x = \frac{21000 \times (1 - 0.8612)}{0.0075}$$

$$\Rightarrow \quad 500000 - x = 388640$$

$$\Rightarrow \quad x = 500000 - 388640$$

$$\Rightarrow \quad x = 111360$$

Hence, the value of x = ₹ 1,11,360.

Amortization of Loans

Amortization refers to the process of paying off a loan through a scheduled, pre-determined sequence of equal payments that includes

(*i*) interest on the outstanding loan and

(*ii*) repayment of part of the loan *i.e.* principal.

A loan is said to be amortized if each installment is used to pay interest and the part of the principal.

Amortization Formulas

While amortizing a loan, one should know the principal outstanding at the beginning of any period, interest and principal amounts in any EMI, total interest paid etc. The formulas for the same are given below:

$$\text{Principal outstanding at beginning of } k\text{th period} = \frac{\text{EMI}\,[(1+i)^{n-k+1} - 1]}{i(1+i)^{n-k+1}} \text{ or EMI} \,.\, a_{\overline{n-k+1}/i}$$

$$\text{Interest paid in } k\text{th payment} = \frac{\text{EMI}\,[(1+i)^{n-k+1} - 1]}{(1+i)^{n-k+1}} \text{ or EMI}\,.\,i.\, a_{\overline{n-k+1}/i}$$

Principal paid in kth payment = EMI – Interest paid in kth period

Total interest paid = $n \times \text{EMI} - \text{P}$

Suppose a person borrowed ₹100000 for 1 year from a bank and wishes to repay the loan in equal monthly installments. Bank charges interest at 12% p.a. compounded monthly. We prepare a table of EMI showing the amount of principal and the amount of interest that comprise each payment until the loan is paid off at the end of its term. This table is known as **amortization schedule**.

Amortization schedule

Month	*Principal outstanding at the beginning of each month*	*EMI paid at the end of each month*	*Interest paid at the end of each month*	*Principal paid at the end of each month*
1	100000	8884.88	1000	7884.88
2	92115.12	8884.88	921.15	7963.73
3	84151.39	8884.88	841.15	8043.37
4	76108.03	8884.88	761.08	8123.80
5	67984.23	8884.88	679.84	8205.04
6	59779.19	8884.88	597.79	8287.09
7	51492.10	8884.88	514.92	8369.96
8	43122.14	8884.88	431.22	8453.66
9	34668.48	8884.88	346.68	8538.20
10	26130.29	8884.88	261.30	8623.58
11	17506.71	8884.88	175.07	8709.81
12	8796.91	8884.88	87.97	8796.91
			6618.50	100000

Example 4. *A couple wishes to purchase a house for ₹1000000 with a down payment of ₹200000. If they can amortize the balance at 9% per annum compounded monthly for 25 years, what is their monthly payment? What is the total interest paid? (Given* $a_{\overline{300}/0.0075} = 119.1616$*)*

Solution. Cost of house = ₹1000000

Down payment = ₹200000

∴ Balance = ₹(1000000 – 200000) = ₹800000.

So, P = ₹800000, $i = \frac{9}{1200} = 0.0075$ and $n = 25 \times 12 = 300$.

$$\therefore \quad \text{EMI} = \frac{P}{a_{\overline{n}/i}} = \frac{800000}{a_{\overline{300}/0.0075}} = \frac{800000}{119.1616} = 6713.57$$

Hence, EMI = ₹6713.57

Now, total interest paid $= n \times \text{EMI} - \text{P}$

$= 300 \times 6713.57 - 800000$

$=$ ₹1214071

Hence, the total interest paid is ₹1214071.

Example 5. *Mr M borrowed ₹ 1000000 from a bank to purchase a house and decided to repay by monthly equal installments in 10 years. The bank charges interest at 9% compounded monthly. The bank calculated his EMI as ₹12668. Find the principal and interest paid in first year. (Given* $a_{\overline{108}/0.0075} = 73.83916$*)*

Solution. Given P = ₹100000, $n = 12 \times 10 = 120$

$i = \frac{9}{1200} = 0.0075$ and EMI = ₹12668.

Principal outstanding at beginning of 13th month = EMI. $a_{\overline{n-k+1}/i}$

$$= 12668.\ a_{\overline{120-13+1}/0.0075}$$

$$= 12668 \times a_{\overline{108}/0.0075}$$

$$= 12668 \times 73.83916$$

$$= ₹935394.48$$

$\therefore$ Principal paid during first year = ₹(1000000 – 935394.48)

$$= ₹64605.52$$

Interest paid during first year = EMI × 12 – 64605.52

$$= 12668 \times 12 - 64605.52$$

$$= 152016 - 64605.52$$

$$= ₹87410.48$$

Example 6. *A loan of ₹250000 at the interest rate of 6% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years, find*

(*i*) *the size of each monthly payment.*

(*ii*) *the principal outstanding at beginning of 40th month.*

(*iii*) *interest paid in 40th payment.*

(*iv*) *principal contained in 40th payment and*

(*v*) *total interest paid.*

(*Given* $(1.005)^{60} = 1.3489$, $(1.005)^{21} = 1.1104$)

Solution. Given, P = ₹250000, $i = \dfrac{6}{12 \times 100} = 0.005$ and $n = 5 \times 12 = 60$.

(*i*) $$\text{EMI} = \frac{250000 \times 0.005 \times (1.005)^{60}}{(1.005)^{60} - 1}$$

$$= \frac{250000 \times 0.005 \times 1.3489}{0.3489} = ₹4832.69$$

(*ii*) Principal outstanding at beginning of 40th month

$$= \frac{\text{EMI}\,[(1+i)^{60-40+1} - 1]}{i\,(1+i)^{60-40+1}} = \frac{4832.69 \times [(1.005)^{21} - 1]}{0.005 \times (1.005)^{21}}$$

$$= \frac{4832.69 \times [1.1104 - 1]}{0.005 \times 1.1104} = \frac{4832.69 \times 0.1104}{0.005 \times 1.1104} = ₹96096.72$$

(*iii*) Interest paid in 40th payment $= \dfrac{\text{EMI}\,[(1+i)^{60-40+1} - 1]}{(1+i)^{60-40+1}}$

$$= \frac{4832.69 \times [(1.005)^{21} - 1]}{(1.005)^{21}} = \frac{4832.69 \times 0.1104}{1.1104} = ₹480.48$$

(*iv*) Principal paid in 40th payment = EMI – Interest paid in 40th payment

$$= 4832.69 - 480.48 = ₹4352.21$$

(*v*) Total interest paid = n × EMI – P = 60 × 4832.69 – 250000

$$= 289961.40 - 250000 = ₹39961.40$$

EXERCISE 12.3

1. Rahul borrowed ₹100000 from a co-operative society at the rate of 10% p.a. for 2 years. Calculate his EMI using flat rate method.
2. Calculate the EMI under 'Flat rate system' for a loan of ₹ 5,00,000 with 10% annual interest rate for 5 years. *(CBSE 2022 Term II)*
3. Jyoti buys a car for which she makes down payment of ₹3,50,000 and the balance is to be paid in 3 years by monthly instalments of ₹34,000 each. If the financer charges interest at the rate of 12% per annum and uses flat rate method, find the actual price of the car.
4. Mrs. Dubey borrowed ₹500000 from a bank to purchase a car and decided to repay by monthly installments in 5 years. The bank charges interest at 8% p.a. compounded monthly. Calculate the EMI. (Given $(1.0067)^{60} = 1.4928$)
5. Utkarsh purchased a laptop worth ₹80000. He paid ₹20000 as cash down and balance in equal monthly installments in 2 years. If bank charges 9% p.a. compounded monthly. Calculate the EMI. (Given $(1.0075)^{24} = 1.1964$)
6. Mr. Bharti wishes to purchase a flat for ₹6000000 with a down payment of ₹1000000 and balance in equal monthly payments for 20 years. If bank charges 7.5% p.a. compounded monthly, calculate the EMI. (Given $(1.00625)^{240} = 4.4608$)
7. Mr. Dharmendra Patel purchased a motorcycle of ₹150000 with ₹25000 down payment. He wishes to repay balance in equal monthly payments in 4 years. If bank charges 9% p.a. compounded monthly, calculate the EMI. (Given $(1.0075)^{48} = 1.4314$)
8. Surjeet purchased a new house, costing ₹40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly installment is ₹30,000 at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him? [Use $(1.0075)^{-300} = 0.1062$]
9. A person amortizes a loan of ₹150000 for a new home by obtaining a 10 year mortgage at the rate of 12% compounded monthly. Find
   (*i*) EMI.
   (*ii*) Total interest paid (Given $a_{\overline{120}/0.01} = 69.6891$).
10. A loan of ₹400000 at the interest rate of 6.75% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years. Find
    (*i*) the size of each monthly payment.
    (*ii*) the principal outstanding at the beginning of 61st month.
    (*iii*) the interest paid in 61st payment.
    (*iv*) the principal contained in 61st payment.
    (*v*) total interest paid.
    (Given $(1.005625)^{120} = 1.9603$, $(1.005625)^{60} = 1.4001$)
11. A person amortizes a loan of ₹1500000 for renovation of his house by 8 years mortgage at the rate of 12% p.a. compounded monthly. Find
    (*i*) the equated monthly installment.
    (*ii*) the principal outstanding at the beginning of 40th month.
    (*iii*) the interest paid in 40th payment.
    (*iv*) the principal contained in 40th payment.
    (*v*) total interest paid.
    (Given $(1.01)^{96} = 2.5993$, $(1.01)^{57} = 1.7633$)

Answers

1. ₹5000 2. ₹12500 3. ₹12,50,000 4. ₹10147.89 5. ₹2741.24
6. ₹40279.70 7. ₹3110.66 8. ₹4,24,800 9. (*i*) ₹2152.42 (*ii*) ₹108290.40
10. (*i*) ₹4593 (*ii*) ₹233336.89 (*ii*) ₹1312.52 (*iv*) ₹3280.48 (*v*) ₹151160
11. (*i*) ₹24379.10 (*ii*) ₹1055326.20 (*iii*) ₹10553.26 (*iv*) ₹13825.84 (*v*) ₹840393.60

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options in questions (1 to 6):

1. The present value of a sequence of payments of ₹800 made at the end of every 6 month and continuing forever, if money is worth 4% p.a. compounded semi-annually, is
(*a*) ₹20000 (*b*) ₹40000 (*c*) ₹60000 (*d*) ₹80000
2. What sum of money invested now could establish a scholarship of ₹2500, which is to be awarded at the end of every year forever, if money is worth 4% compounded annually?
(*a*) ₹62500 (*b*) ₹125000 (*c*) ₹31250 (*d*) none of these
3. At what rate of interest will the present value of a perpetuity of ₹500 payable at the end of each quarter be ₹40000?
(*a*) 1.25% p.a. (*b*) 2.5% p.a. (*c*) 5% p.a. (*d*) 6% p.a.
4. The present value of a perpetuity of ₹750 payable at the beginning of each year, if money is worth 5% p.a., is
(*a*) ₹15000 (*b*) ₹15750 (*c*) ₹14250 (*d*) none of these
5. What amount should be deposited at the end of every 6 months to accumulate ₹50000 in 8 years, if money is worth 6% p.a. compounded semi-annually? (Given $(1.03)^{16} = 1.6047$)
(*a*) ₹3432.53 (*b*) ₹2783.08 (*c*) ₹2480.57 (*d*) ₹2149.93
6. Mr. X borrowed ₹ 500000 from a bank to purchase a house and decided to repay the loan by equal monthly payments in 10 years. If bank charges interest at 7.5% p.a. compounded monthly, then EMI is (Given $(1.00625)^{120} = 2.1121$)
(*a*) ₹5935 (*b*) ₹6380 (*c*) ₹7340 (*d*) ₹8520

Answers

1. (*b*) 2. (*a*) 3. (*c*) 4. (*b*) 5. (*c*) 6. (*a*)

Assertion-Reason Type Questions (Solved)

These type of questions consist of two Statements.

Statement I is called Assertion (A) and Statement II is called Reason (R). Read the given Statements carefully and choose the correct answer from the options given below:

(*a*) *Both the Statements are true and Statement II is the correct explanation of Statement I.*
(*b*) *Both the Statements are true and Statement II is not the correct explanation of Statement I.*
(*c*) *Statement I is true, Statement II is false.*
(*d*) *Statement I is false, Statement II is true.*

1. ***Statement I:*** *The present value of a perpetuity of ₹20000 payable at the beginning of every quarter is ₹16,20,000 if the money is worth 5% per annum compounded quarterly.*

   Statement II: *The present value of a perpetuity is* $P = \frac{R}{i}$, *when a periodic payment of ₹R is made at the beginning of each payment period and interest being ₹i per rupee per period.*

2. ***Statement I:*** *The present value of a perpetuity whose periodic payment of ₹R is made at the end of each payment period, interest being ₹i per rupee per period is given by* $\frac{R}{i}$.

   Statement II: *Sum of an infinite G.P. whose first term is a and common ratio is r (r < 1) is given by* $\frac{a}{1-r}$.

3. ***Statement I:*** *The EMI under flat rate method for a loan of ₹1,00,000 at the rate of 12% per annum for 4-years is ₹3083.33.*

   Statement II: *Under flat rate method of calculating EMI, the borrower is at loss because the interest remains constant for every EMI and does not reduce as the outstanding principal amount decreases with every EMI.*

4. ***Statement I:*** *Under reducing balance method,*

   principal outstanding at beginning of kth period $= \frac{\text{EMI}\,[(1+i)^{n-k+1}-1]}{i\,(1+i)^{n-k+1}}$

   and interest paid in kth payment $= \frac{\text{EMI}\,[(1+i)^{n-k+1}-1]}{(1+i)^{n-k+1}}$

   where i = interest rate per rupee per month and n = number of payments.

   Statement II: *Interest on P at the rate of i per rupee per period is given by P × i for one period.*

Solutions to Assertion-Reason Type Questions

1. Given R = ₹20,000, $i = \frac{5}{4 \times 100} = 0.0125$

   So, present value $P = R + \frac{R}{i}$ = ₹20,000 + $\frac{20000}{0.0125}$

   = ₹20,000 + ₹16,00,000 = ₹16,20,000.

   ∴ Statement I is true

   Statement II is false.

   ∴ Option (*c*) is the correct answer.

2. Both Statements I and II are true and Statement II is the correct explanation of Statement I.

   ∴ Option (*a*) is the correct answer.

3. Given P = ₹1,00,000, $i = \frac{12}{12 \times 100} = 0.01$ and $n = 12 \times 4 = 48$ months.

   So, EMI $= \frac{P + Pni}{n} = \frac{100000 + 100000 \times 48 \times 0.01}{48}$

   $= \frac{100000 + 48000}{48} = \frac{148000}{48}$ = ₹3083.33

   ∴ Statement I is true.

   Also, Statement II is true.

   Statement II is not the correct explanation of Statement I.

   ∴ Option (*b*) is the correct answer.

4. Interest paid in kth payment = (Principal outstanding at the beginning of kth period) × i

$$= \frac{\text{EMI }[(1+i)^{n-k+1}-1]}{i[(1+i)^{n-k+1}]} \times i = \frac{\text{EMI }[(1+i)^{n-k+1}-1]}{(1+i)^{n-k+1}}.$$

Hence, both the Statements I and II are true and Statement II is the correct explanation of Statement I.

∴ Option (*a*) is the correct answer.

Assertion-Reason Type Questions (Unsolved)

These type of questions consist of two Statements.

Statement I is called Assertion (A) and Statement II is called Reason (R). Read the given Statements carefully and choose the correct answer from the options given below:

(*a*) Both the Statements are true and Statement II is the correct explanation of Statement I.

(*b*) Both the Statements are true and Statement II is not the correct explanation of Statement I.

(*c*) Statement I is true, Statement II is false.

(*d*) Statement I is false, Statement II is true.

1. **Statement I:** The present value of a perpetuity of ₹5000 payable at the end of every month is ₹10,00,000, if the money is worth 6% p.a. compounded monthly.

   Statement II: Present value of a perpetuity is $P = \frac{R}{i}$,

   where a periodic payment of ₹R is made at the end of each payment period and interest being ₹ i per rupee per period.

2. **Statement I:** Periodic payments of ₹ R payable at the end of each period for n periods interest being ₹ i per rupee per period amounts to ₹ $R\left[\frac{(1+i)^n - 1}{i}\right]$.

   Statement II: ₹1 amounts to ₹$(1+i)^n$ after n periods, interest being ₹1 per rupee period.

3. **Statement I:** In sinking fund a fixed amount at regular intervals is deposited.

   Statement II: In savings account any amount, any time can be deposited.

4. **Statement I:** Mr Jain borrowed ₹8,00,000 for the renovation of his house and decided to repay the loan by equal monthly installments in 15 years.

   If bank charges interest at 7.5% p.a. compounded monthly, then EMI is ₹7416. (Given $(1.00625)^{180} = 3.0695$)

   Statement II: $\text{EMI} = \frac{P \times (1+i)^n}{(1+i)^n - 1}$

   where P = Principal borrowed, i = interest rate per rupee per month and n = number of payments.

5. **Statement I:** In reducing balance method, the interest is calculated on the principal outstanding.

   Statement II: EMI has two parts principal and interest on outstanding principal. As the loan is paid off with every EMI, outstanding principal decreases and in subsequent EMI's principal paid increases and interest portion decreases.

Answers

1. (*a*)	2. (*a*)	3. (*b*)	4. (*c*)	5. (*a*)

CASE-STUDY BASED QUESTION (UNSOLVED)

In year 2000, Mr. Talwar took a home loan of ₹3000000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions:

(*a*) Find the equated monthly installment paid by Mr. Talwar.

(*b*) Find the interest paid by Mr. Talwar in 150th payment.

OR

Find the principal paid by Mr. Talwar in 150th payment.

(*c*) Find the total interest paid by Mr. Talwar.

[Use $(1.00625)^{240} = 4.4608$, $(1.00625)^{91} = 1.7629$, $(1.00625)^{48} = 1.1187$]

Answer

(*a*) ₹24167.82 (*b*) ₹10458.69 **OR** ₹13709.13 (*c*) ₹2800276.80