13 Returns, Growth and Depreciation

INTRODUCTION

Most of us invest money in different schemes and expect returns from it. There are many companies in the market which attract people to invest in their investment plans. We must evaluate the investment based on our financial needs. The return on investment helps us choose the best investment plan across different investment options. In this chapter, we shall learn to calculate rate of return, nominal rate of return, effective rate of return, compound annual growth rate and depreciation.

RETURN ON INVESTMENT (ROI)

Return on investment is used to measure the efficiency of an investment or to know the efficiency of a number of investments. Return on investment directly measure the amount of return on a particular investment relative to the investment cost. It is expressed as the ratio

$$\text{R.O.I} = \frac{\text{Net profit/loss}}{\text{Cost of investment}}$$

$$\text{Or, R.O.I} = \frac{\text{Current value of investment} - \text{Cost of investment}}{\text{Cost of investment}}.$$

It can also be expressed as a percentage, then it is termed as **rate of return.**

Rate of return is the gain or loss of an investment over a certain period of time. Rate of return can be calculated by using the formula

$$\text{Rate of return} = \frac{\text{Net profit/loss}}{\text{Cost of investment}} \times 100$$

$$\text{Or, rate of return} = \frac{\text{Current value of investment} - \text{Cost of investment}}{\text{Cost of investment}} \times 100.$$

Rate of return shows us the profitability of our investment. Rate of return may be positive or negative. If rate of return is negative, it means we are in loss and if rate of return is positive, it means we are in profit.

ILLUSTRATIVE EXAMPLES

Example 1. *A man invests ₹9600 on ₹100 shares at ₹80. If the company pays him 18% dividend, find his rate of return.*

Solution. Cost of investment = ₹9600

Market value of one share = ₹80

$$\therefore \quad \text{Number of shares bought} = \frac{₹9600}{₹80} = 120$$

Total dividend earned by man = No. of shares × rate of dividend × face value of one share

$$= ₹(120 \times \frac{18}{100} \times 100) = ₹2160$$

So, rate of return = $\frac{2160}{9600} \times 100\% = 22.5\%$

Example 2. *A person has an initial investment of ₹50000 in an investment plan. After 2 years it has grown to ₹60000. Find his rate of return.*

Solution. Given current value of investment = ₹60000

Cost of investment = ₹50000

$\therefore$ Rate of return = $\frac{60000 - 50000}{50000} \times 100\% = 20\%$

Investment in share market involves the receipt of dividends and a capital gain or loss. Suppose a stock is held for one year, then the rate of return can be calculated as

$$\text{Rate of return} = \frac{D_1 + (P_1 - P_0)}{P_0} \times 100\%$$

where P_0 = Price of shares/stocks at the time of investment

P_1 = Price of shares/stocks at the end of the year

D_1 = Dividend earned during the year.

Example 3. *Mr Parekh invested ₹52000 on ₹100 shares at a discount of ₹20 paying 8% dividend. At the end of the year he sells the shares at a premium of ₹20. Find his rate of return.*

Solution. Market value of one share = ₹100 – ₹20 = ₹80

Cost of investment = ₹52000

$\therefore$ Number of shares bought = $\frac{₹52000}{₹80} = 650.$

Annual dividend earned = No. of shares × rate of dividend × face value of one share

$$= ₹650 \times \frac{8}{100} \times 100 = ₹5200.$$

As Mr Parekh sells his shares at a premium of ₹20

So, market value of one share = ₹100 + ₹20 = ₹120.

$\therefore$ The selling value of 650 shares = ₹120 × 650 = ₹78000.

Here, P_0 = ₹52000, D_1 = ₹5200, P_1 = ₹78000.

$\therefore$ Rate of return = $\frac{D_1 + (P_1 - P_0)}{P_0} \times 100\% = \frac{5200 + (78000 - 52000)}{52000} \times 100\%$

$$= \frac{31200}{52000} \times 100\% = 60\%$$

NOMINAL RATE OF RETURN

Nominal rate of return is the total amount of money earned by an investment without considering various expenses such as taxes, legal fees, inflation etc. It is the basic return offered by the investment. So, all the formulae we have learnt in this chapter till now are the formulae of calculating the nominal rate of return.

The actual rate of return can be calculated by subtracting the various expenses from the total money earned, which is relatively less than nominal rate of return. The calculation of actual rate of return is beyond the scope of this book.

EXERCISE 13.1

1. A person invests ₹10000 in 10% ₹100 shares of a company available at a premium of ₹25. Find his rate of return.
2. A man invests ₹22500 in ₹50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate his rate of return.
3. A person invested ₹200000 in a fund for one year. At the end of the year, the investment was worth ₹216000. Calculate his rate of return.
4. Mitul invested ₹3,50,000 in a fund. At the end of the year the value of the fund is ₹4,37,500. What is the nominal rate of interest, if the market price is same at the end of the year?
5. Salmaan invests a sum of money in ₹50 shares paying 10% dividend quoted at 20% discount. If his annual dividend is ₹600, calculate his rate of return from the investment.
6. A man invests a sum of money in ₹100 shares paying 15% dividend quoted at 20% premium. If his annual dividend is ₹540, calculate the rate of return on his investment.
7. Mr Satendra holds 1500, ₹100 shares of a company paying 15% dividend annually quoted at 30% premium. Calculate rate of return on his investment.
8. ₹100 shares of a company are sold at a discount of ₹20. If the return on the investment is 15%, find the rate of dividend declared.
9. A company declared a dividend of 14%. Find the market value of ₹50 shares, if the return on the investment was 10%.
10. Mrs Gupta invested ₹16500 on ₹100 shares at a premium of ₹10 paying 15% dividend. At the end of the year, she sells the shares at a premium of ₹20. Find her rate of return.
11. Mr Verma invested ₹18000 on ₹100 shares at a discount of ₹25 paying 12% dividend. At the end of the year, he sells the shares at a discount of ₹10. Find his rate of return.

Answers

1. 8%	2. $13\frac{1}{3}\%$	3. 8%	4. 25%	5. 12.5%	6. 12.5%
7. $11\frac{7}{13}\%$	8. 12%	9. ₹70	10. $22\frac{8}{11}\%$	11. 36%	

EFFECTIVE RATE OF RETURN

Effective rate of return is the real rate of return generated by an investment when all factors effecting returns are considered. These factors are as follows:

(*i*) **Price at which the investment is done.**

The rate of return is effected by the price at which the investment is done. If an investor purchases shares/stocks at less than face value of the shares, then the effective rate of return increases and if the investor purchases shares/stocks at more than face value of the shares then the effective rate of return decreases.

For example, if a 8% bond of face value ₹1000 is purchased at ₹800, then actual rate of return will be 10%. On the other hand, if 8% bond of face value ₹1000 is purchased at ₹1050, then actual rate of return will be 7.62% (approx.)

(*ii*) **Stated or declared interest rate.**

The rate of return is indirectly effected by the stated or declared interest rate. The effect is seen only when compounding is done more than once in a year. If the compounding is not done then actual rate of return is same as the stated or declared rate.

For example, consider an amount of ₹10000 invested at 4% interest compounded quarterly. So, the amount at the end of one year = ₹10000 $(1.01)^4$ = ₹10406. This is equivalent to interest of 4.06% compounded annually because ₹10000 (1.0406) = ₹10406. In the above example effective rate is 4.06%.

The formula of calculating the effective rate is

$$\text{Effective rate of return (per rupee)} = \left(1+\frac{r}{100p}\right)^{p} - 1$$

where r = annual rate of interest

p = number of compounding in a year.

If interest is compounded continuously, then

$$\text{effective rate of return (per rupee)} = \lim_{p\to\infty}\left[\left(1+\frac{r}{100p}\right)^{p} - 1\right]$$

$$= \lim_{p\to\infty}\left[\left(1+\frac{r}{100p}\right)^{\frac{100p}{r}}\right]^{\frac{r}{100}} - 1$$

As $p \to \infty$, $\frac{r}{100p} \to 0$.

So, effective rate of return (per rupee)

$$= \lim_{\frac{r}{100p}\to 0}\left[\left(1+\frac{r}{100p}\right)^{\frac{100p}{r}}\right]^{\frac{r}{100}} - 1$$

$$= e^{\frac{r}{100}} - 1 \qquad \left[\because \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right]$$

$\therefore$ Effective rate of return (per rupee) $= e^{i} - 1$, where $i = \frac{r}{100}$.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the effective rate which is equivalent to a stated rate of 6% compounded semiannually.*

Solution. Given $r = 6\%$ p.a.

$p = 2$ half years.

So, effective rate (per rupee) $= \left(1+\frac{6}{200}\right)^{2} - 1 = (1.03)^2 - 1$

$= (1.0609) - 1 = 0.0609$

Hence, the effective rate $= (0.0609) \times 100\% = 6.09\%$

Example 2. *Mr Dinesh has two investment options either 10% per annum compounded semiannually or 9.8% per annum compounded quarterly. Which option is better for Mr Dinesh? Given* $(1.0245)^4 = 1.1017$.

Solution. ***First option.*** Given $r = 10\%$ p.a., $p = 2$ half years

So, effective rate (per rupee) $= \left(1+\frac{10}{200}\right)^{2} - 1 = (1.05)^2 - 1$

$= 1.1025 - 1 = 0.1025$

Thus, effective rate $= (0.1025) \times 100\% = 10.25\%$

Second option. Given $r = 9.8\%$ p.a., $p = 4$ quarters

So, effective rate (per rupee) $= \left(1+\frac{9.8}{400}\right)^4 - 1 = (1.0245)^4 - 1$

$= 1.1017 - 1 = 0.1017$

Thus, effective rate $= 0.1017 \times 100\% = 10.17\%$

Hence, the first option is better for Mr Dinesh.

Example 3. *Mr X took a loan of ₹2000 for 6 months. Lender deducts ₹200 as interest while lending. Find the effective rate of interest charged by lender.*

Solution. Since the moneylender deducted ₹200 while lending a loan of ₹2000 for 6 months, therefore, ₹200 can be treated as interest on ₹1800 for 6 months.

So, interest rate per 6 months $= \frac{200}{1800} = \frac{1}{9}$.

$\therefore$ Effective rate of interest (per rupee) $= \left(1+\frac{1}{9}\right)^2 - 1$

$= \frac{100}{81} - 1 = 1.23456 - 1$

$= 0.2346$

Hence, effective rate $= (0.2346) \times 100\% = 23.46\%$

Example 4. *Find the effective rate of return equivalent to a nominal rate of 6% per annum compounded*

(*i*) *quarterly* (*ii*) *continuously. (Given* $(1.015)^4 = 1.06136$ *and* $e^{0.06} = 1.06183$*)*

Solution. (*i*) Given $r = 6\%$ p.a.

$p = 4$ quarters.

So, effective rate (per rupee) $= \left(1+\frac{6}{400}\right)^4 - 1 = (1.015)^4 - 1$

$= 1.06136 - 1 = 0.06136 = 0.0614$

Hence, effective rate $= 0.0614 \times 100\% = 6.14\%$

(*ii*) Given $r = 6\%$ p.a.

So, $i = \frac{6}{100} = 0.06$

When compounded continuously, then

effective rate (per rupee) $= e^i - 1 = e^{0.06} - 1$

$= 1.06183 - 1 = 0.06183 = 0.0618$

Hence, effective rate $= 0.0618 \times 100\% = 6.18\%$

COMPOUND ANNUAL GROWTH RATE (CAGR)

Suppose Rahul invested ₹100000 in the stocks of a company for 5 years. The value of his investment at the end of each year is given below:

Year 1	₹108000
Year 2	₹87000
Year 3	₹95000
Year 4	₹104000
Year 5	₹117000

Rahul wants to know a steady annual growth rate of his investment *i.e.* on average what annual rate of return he is getting on his investment. This steady annual growth rate is called compound annual growth rate abbreviated as CAGR.

Compound annual growth rate is a measure of an average yearly growth of an investment over a certain time period when returns/profits are reinvested at the end of each year. CAGR is very useful for investors to compare different investment plans. It is an accurate measure of investment growth over time period.

The formula for calculating CAGR can be obtained from the compound interest formula which you have learnt in lower classes.

$$A = P(1 + i)^n \text{ or } F.V. = P.V.\ (1 + i)^n$$

where F.V. = Future/Final value of an investment

P.V. = Present/Beginning value of an investment

$i = \frac{r}{100}$, r = rate of interest per rupee p.a.

n = number of years.

$$\text{So, } \frac{F.V.}{P.V} = (1 + i)^n$$

$$\Rightarrow \quad 1 + i = \left(\frac{F.V.}{P.V.}\right)^{\frac{1}{n}} \Rightarrow i = \left(\frac{F.V.}{P.V.}\right)^{\frac{1}{n}} - 1$$

where, i = compound annual growth rate.

$$\text{So, CAGR} = \left(\frac{F.V.}{P.V}\right)^{\frac{1}{n}} - 1.$$

Example 5. *Diwakar invested ₹20000 in a mutual fund in year 2010. The value of mutual fund increased to ₹32000 in year 2015. Calculate the compound annual growth rate of his investment.*

Solution. Given beginning value of the investment = ₹20000

Final value of the investment = ₹32000

Number of years = 5

$$\text{So, CAGR} = \left(\frac{F.V.}{P.V}\right)^{\frac{1}{n}} - 1 = \left(\frac{32000}{20000}\right)^{\frac{1}{5}} - 1 = (1.6)^{\frac{1}{5}} - 1$$

Let $x = (1.6)^{\frac{1}{5}}$

$$\Rightarrow \quad \log x = \frac{1}{5} \log 1.6 = \frac{1}{5} \times 0.2041 = 0.04082$$

$$\Rightarrow \quad x = \text{antilog } 0.04082 = 1.098$$

So, CAGR = 1.098 – 1 = 0.098

Hence, CAGR = 0.098 × 100% = 9.8%

Example 6. *Vikas invested ₹10000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below:*

Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Year 6*
₹11000	*₹11500*	*₹13000*	*₹11800*	*₹12200*	*₹14000*

Calculate CAGR of his investment.

Solution. Given P.V. = ₹10000, F.V. = ₹14000, n = 6 years.

So, CAGR = $\left(\frac{14000}{10000}\right)^{\frac{1}{6}} - 1 = (1.4)^{\frac{1}{6}} - 1$

Let $x = (1.4)^{\frac{1}{6}}$

$\Rightarrow \log x = \frac{1}{6} \log 1.4 = \frac{1}{6} \times 0.1461 = 0.02435$

$\Rightarrow \log x = 0.0244$

$\Rightarrow x = \text{antilog } 0.0244$

$\Rightarrow x = 1.058$

CAGR = 1.058 – 1 = 0.058

Hence, CAGR = 0.058 × 100% = 5.8%

Example 7. *Suppose a person invested ₹15000 in a mutual fund and the value of investment at the time of redemption was ₹25000. If CAGR for this investment is 8.88%. Calculate the number of years for which he has invested the amount.*

Solution. Given P.V. = ₹15000, F.V. = ₹25000, CAGR = 8.88%

$\because \quad \text{CAGR\%} = \left[\left(\frac{\text{F.V.}}{\text{P.V.}}\right)^{\frac{1}{n}} - 1\right] \times 100$

$\Rightarrow \quad 8.88 = \left[\left(\frac{25000}{15000}\right)^{\frac{1}{n}} - 1\right] \times 100$

$\Rightarrow \quad 0.0888 + 1 = (1.667)^{\frac{1}{n}}$

$\Rightarrow \quad 1.0888 = (1.667)^{\frac{1}{n}}$

Taking logarithm on both sides, we get

$\log 1.0888 = \frac{1}{n} \log 1.667$

$\Rightarrow \quad n = \frac{\log 1.667}{\log 1.089} = \frac{0.2219}{0.0370} = 5.99$ *i.e.* 6 years.

Difference between Average Annual Growth Rate (AAGR) and Compound Annual Growth Rate (CAGR)

Average annual growth rate (AAGR)	Compound annual growth rate (CAGR)
AAGR is the average annualized return of an investment.	CAGR is compounded annual return of an investment.
AAGR is calculated by taking arithmetic mean of series of returns.	CAGR is calculated by using the final and beginning value of an investment.
AAGR is linear measure that does not account for the effects of compounding.	CAGR describes the effect of compounding. It smoothens out the volatile nature of year by year growth/decay rates and provides more accurate results.

USE OF COMPOUND ANNUAL GROWTH RATE

CAGR can be used to

(*i*) calculate and communicate the average growth of a single investment.

(*ii*) demonstrate and compare the performance of investment advisors.

(*iii*) compare the historical returns of stocks with bonds or with a savings account.

(*iv*) track the performance of various business measures of one or multiple companies alongside one another.

EXERCISE 13.2

1. Find the effective rate of return which is equivalent to a stated rate of 8% compounded quarterly. [Use $(1.02)^4 = 1.0824$]
2. Find the declared rate of return compounded semiannually which is equivalent to 6% effective rate of return. [Use $(1.06)^{1/2} = 1.0296$]
3. Find the effective rate which is equivalent to nominal rate of 10% p.a. compounded monthly.

   [Given that : $(1.00833)^{12} = 1.1047$]
4. Find the effective rate of return equivalent to declared rate of 12% compounded *(CBSE 2022 Term II)*

   (*i*) semiannually (*ii*) quarterly (*iii*) monthly (*iv*) continuously

   [Use $(1.06)^2 = 1.1236$, $(1.03)^4 = 1.1255$ and $(1.01)^{12} = 1.1268$, $e^{0.12} = 1.1275$]
5. Mrs Renu has two investment options 8.4% p.a. compounded monthly or 8.5% p.a. compounded semiannually. Which option is better for Mrs Renu?

   [Use $(1.007)^{12} = 1.0873$, $(1.0425)^2 = 1.0868$]
6. Mr Deepanshu wants to make an investment of ₹10000 for 5 years. He has two options. First option fetches him a return of 8% compounded semiannually and second option fetches him a return of 7.5% compounded quarterly. Which investment option should he go for?

   [Use $(1.04)^2 = 1.0816$ and $(1.01875)^4 = 1.0771$]
7. Mrs Vandana invested ₹35000 in a shares of a company and reinvested the earnings every year in buying the shares of the same company. At the end of 5 years, the value of shares increased to ₹56000. Calculate the compound annual growth rate of her investment.
8. Mr Mohan invested ₹5000 in a fund at the beginning of year 2018 and by the end of year 2018 his investment was worth ₹9000. Next year market crashed and he lost ₹3000 and ending up with ₹6000 at the end of year 2019. Next year *i.e.* 2020 he gained ₹4500 and ending up with ₹10500 at the end of the year. Find CAGR of his investment.
9. Bhandari invested ₹10000 in a portfolio with return given below:

   From April 1, 2012 to March 31, 2013 portfolio grew to ₹12000

   From April 1, 2013 to March 31, 2014 portfolio grew to ₹12500

   From April 1, 2014 to March 31, 2015 portfolio grew to ₹13800

   On March 31, 2016 the portfolio ended with ₹18000.

   Calculate the compound annual growth rate of portfolio.
10. Mr Tripathi invested ₹10000 in a company's fund. His yearly investment values are shown in the table given below:

Year	0	1	2	3
Amount (in ₹)	10000	13000	11000	9400

Calculate CAGR of his investment.

11. Mr Kumar has invested ₹20000 in year 2014 for 5 years. If CAGR for that investment turned out to be 11.84%. What will be the end balance?

12. Mr Naresh has bought 200 shares of city look company at ₹100 each in 2015. After selling them he has received ₹30000 which accounts for 22.47% CAGR. Calculate the number of years for which he was holding the shares.

13. A start-up company invested ₹ 3,00,000 in shares for 5 years. The value of this investment was ₹ 3,50,000 at the end of second year, ₹ 3,80,000 at the end of third year and on maturity, the final value stood at ₹ 4,50,000. Calculate the compound Annual Growth Rate (CAGR) on the investment.

[Given that : $(1.5)^{\frac{1}{5}} = 1.084$] *(CBSE 2022 Term II)*

Answers

1. 8.24%	**2.** 5.92%	**3.** 10.47%			
4. (*i*) 12.36%	(*ii*) 12.55%	(*iii*) 12.68%	(*iv*) 12.75%	**5.** compounded monthly	
6. First option	**7.** 9.8%	**8.** 28%	**9.** 15.8%	**10.** −2.05%	**11.** ₹35000
12. 2 years	**13.** 8.4%				

DEPRECIATION

Depreciation is defined as the reduction of original cost of a fixed asset such as buildings, furniture, machinery, vehicles etc. over a period of time due to wear and tear. **Original cost** is the total price at which the asset was purchased.

The time period in which an asset is expected to be functional and fit for purpose is called **useful life**. It is generally measured in years.

The value of a depreciable asset at the end of its useful life is called **scrap value** or **salvage value** or **depreciated value**.

Book value. Book value of an asset at a given date is the original value of the asset minus the accumulated depreciation at that date.

Methods of Computing Depreciation

There are several methods of computing depreciation. The most common methods are given below:

(*i*) Linear method or straight line method

(*ii*) Declining balance method or written down value method

(*iii*) Sum of years digits method

(*iv*) Units of production method

Here, we shall learn linear method of depreciation, declining balance method of depreciation and sum of years digit method.

(i) Linear or Straight Line Method

The linear or straight line depreciation method is the simplest and most widely used method. In this method, the value of an asset is reduced uniformly over each period until it reaches to its scrap or salvage value. The formula of calculating annual depreciation by linear method is given below:

$$\text{Annual depreciation} = \frac{\text{Original cost of an asset} - \text{Scrap value}}{\text{Useful life of asset in years}}.$$

ILLUSTRATIVE EXAMPLES

Example 1. *A machine costing ₹50000 has a useful life of 4 years. The estimated scrap value is ₹10000. Using straight line method, find the annual depreciation.*

Solution. Given original cost of machine = ₹50000

Scrap value of machine = ₹10000

Useful life = 4 years

$$\therefore \text{Annual depreciation} = \frac{50000 - 10000}{4} = \frac{40000}{4} = 10000$$

Hence, annual depreciation is ₹10000.

Example 2. *The annual depreciation of a car is ₹30000. If the scrap value of the car after 15 years is ₹50000, find the original cost of the car using linear method.*

Solution. Given annual depreciation = ₹30000

Scrap value of car = ₹50000

Useful life = 15 years

Let the original cost of the car be ₹C, then using

$$\text{annual depreciation} = \frac{\text{original cost} - \text{scrap value}}{\text{useful life}}, \text{ we get}$$

$$30000 = \frac{C - 50000}{15}$$

$\Rightarrow$ $C - 50000 = 450000$

$\Rightarrow$ $C = 450000 + 50000 = 500000.$

Hence, the original cost of the car was ₹500000.

Example 3. *A dining table costing ₹36000 has a useful life of 15 years. If annual depreciation is ₹2000, find its scrap value using linear method.*

Solution. Given original cost of dining table = ₹36000

Useful life = 15 years

Annual depreciation = ₹2000

Let the scrap value of the dining table be ₹S, then using

$$\text{annual depreciation} = \frac{\text{original cost} - \text{scrap value}}{\text{useful life}}, \text{ we get}$$

$$2000 = \frac{36000 - S}{15}$$

$\Rightarrow$ $30000 = 36000 - S \Rightarrow S = 6000$

Hence, the scrap value of dining table is ₹6000.

EXERCISE 13.3

1. A piece of equipment that costs ₹25000 has an estimated useful life of 8 years and ₹0 scrap value. Find the annual depreciation using linear method.
2. A machine costing ₹30000 is expected to have a useful life of 13 years and a final scrap value of ₹4000. Find the annual depreciation charge using straight line method.
3. An asset the life of which is estimated to be 8 years costs ₹80000. If annual depreciation is ₹9000, find its scrap value using linear method.
4. A vehicle costing ₹900000 has a scrap value of ₹270000. If annual depreciation charge is ₹70000, find its useful life in years.

5. A machine has a scrap value of ₹22500 after 15 years of its purchase. If the annual depreciation charge is ₹8500, find its original cost using linear method.
6. Abhay bought a mobile phone for ₹ 30,000. The mobile phone is estimated to have a scrap value of ₹ 3000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years. *(CBSE 2022 Term II)*

Answers

1. ₹3125 2. ₹2000 3. ₹8000 4. 9 years 5. ₹150000 6. ₹12000

(ii) Declining Balance Method or Written Down Value Method

Declining balance method is one of the most common method used by companies. This method is also known as reducing balance method. A fixed percentage of depreciation is calculated each year to the depreciated value of an asset at the end of the previous year, which does not remain constant but keeps on reducing every year.

The formula of calculating the depreciated value or scrap value of an asset is given below:

$$S = P(1 - i)^n$$

where S = Depreciated or scrap value of an asset

P = Present value of an asset

$i = \frac{r}{100}$, r = constant rate of depreciation

n = number of years.

ILLUSTRATIVE EXAMPLES

Example 1. *The value of a machine purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹97200, find*

(i) its value after 3 years.

(ii) its value when it was purchased.

Solution. Given P = ₹97200, r = 10% p.a.

$$\Rightarrow \quad i = \frac{10}{100} = 0.1$$

(i) Value after 3 years = present value $(1 - 0.1)^3$

$$= 97200 \times (0.9)^3 = 97200 \times 0.729$$

$$= ₹70858.80$$

(ii) Present value = value 2 years ago $\times (1 - 0.1)^2$

$$\Rightarrow \quad ₹97200 = \text{value 2 years ago} \times (0.9)^2$$

$$\Rightarrow \quad \text{value 2 years ago} = \frac{₹97200}{0.81} = ₹120000$$

Example 2. *The cost of a washing machine depreciates by ₹720 during the second year and by ₹648 during the third year. Calculate:*

(i) the rate of depreciation per annum.

(ii) the original cost of the machine.

(iii) the value of the machine at the end of third year.

Solution. (i) Let the original cost of the washing machine be ₹P and the rate of depreciation be r% p.a. Then the value of machine (in ₹) after one year, two years and 3 years are $P(1 - i)$, $P(1 - i)^2$ and $P(1 - i)^3$ respectively, where $i = \frac{r}{100}$.

According to given,

$P(1-i) - P(1-i)^2 = 720$ and $P(1-i)^2 - P(1-i)^3 = 648$

$\Rightarrow P(1-i)[1-(1-i)] = 720$ and $P(1-i)^2[1-(1-i)] = 648$

$\Rightarrow P(1-i)\,i = 720$ ...(1) and $P(1-i)^2.i = 648$ ...(2)

Dividing (2) by (1), we get

$$1-i = \frac{648}{720} \Rightarrow 1-i = \frac{9}{10}$$

$$\Rightarrow i = 1 - \frac{9}{10} \Rightarrow i = \frac{1}{10} \Rightarrow \frac{r}{100} = \frac{1}{10}$$

$\Rightarrow r = 10$

Hence, the rate of depreciation = 10% p.a.

(*ii*) Putting $i = \frac{1}{10}$ in equation (1), we get

$$P\left(1-\frac{1}{10}\right)\times\frac{1}{10} = 720$$

$$\Rightarrow P \times \frac{9}{100} = 720 \Rightarrow P = 8000$$

Hence, the original cost of the machine = ₹8000

(*iii*) The value of machine at the end of third year $= P(1-i)^3$

$$= 8000\left(1-\frac{1}{10}\right)^3 = 8000\,(0.9)^3$$

$$= 8000 \times 0.729 = 5832$$

Hence, the value of machine at the end of third year = ₹5832.

Example 3. *The value of a car depreciates by 12.5% every year. By what percent will the value of the car decrease after 3 years?*

Solution. Let the present value of the car be ₹P, then

value of the car after 3 years $= P(1-i)^3 = P\left(1-\frac{12.5}{100}\right)^3$

$$= P\left(1-\frac{1}{8}\right)^3 = P\left(\frac{7}{8}\right)^3$$

Decrease in the value of car $= P - P\left(\frac{7}{8}\right)^3 = P\left[1-\frac{343}{512}\right] = P\times\frac{169}{512}$.

$\therefore$ Percentage decrease in the value of the car after 3 years

$$= \left(\frac{\text{Decrease in value}}{\text{Present value}}\times 100\right)\% = \left(\frac{P\times\frac{169}{512}}{P}\times 100\right)\%$$

$$= \frac{169\times 25\%}{128} = \frac{4225}{128}\% = 33\frac{1}{128}\%$$

(iii) Sum of the Years Digit Method

In this method, the fraction of the asset to be depreciated each year is obtained by putting the digits of the years in reverse order over the sum of the years digits. A greater fraction of the cost of the asset is depreciated in the earlier years of the life of the asset.

Example 4. *An asset costing ₹10000 is expected to have a useful life of 4 years and a scrap value of zero. Find the annual depreciation charge using the sum of the years digits method.*

Solution. Given that original cost of an asset = ₹10000, useful life = 4 years.

Sum of the years digits = 1 + 2 + 3 + 4 = 10.

The annual depreciation charged each year is determined by putting the digits of the year in reverse order over the sum of the years digits *i.e.* 10.

Depreciation Schedule

Year	Digit of the year in reverse order	Fraction of the asset to be depreciated	Annual depreciation (in ₹)	Accumulated depreciation (in ₹)
1	4	$\frac{4}{10}$	$\frac{4}{10} \times 10000 = 4000$	4000
2	3	$\frac{3}{10}$	$\frac{3}{10} \times 10000 = 3000$	7000
3	2	$\frac{2}{10}$	$\frac{2}{10} \times 10000 = 2000$	9000
4	1	$\frac{1}{10}$	$\frac{1}{10} \times 10000 = 1000$	10000

EXERCISE 13.4

1. The cost of a mobile phone is ₹9000. Its value depreciates at the rate of 5% p.a. Find the total depreciation in its value at the end of 2 years.
2. Dinesh purchases a washing machine for ₹24000. The value of machine is depreciating at the rate of 5% p.a. Calculate its value after 3 years.
3. Rahul purchased an old scooter for ₹16000. If the cost of the scooter after 2 years depreciates to ₹14440, find the rate of depreciation.
4. The cost of a car purchased 2 years ago, depreciates at the rate of 20% every year. If its present worth is ₹315600, find:
   (*i*) its purchase price (*ii*) its value after 3 years.
5. A machine costing ₹50000 depreciates at a constant rate of 8% p.a. If the estimated useful life of the machine is 10 years, determine its scrap value (Use log table).
6. The cost of a machine depreciating by ₹722 during the third year and by ₹685.90 during the fourth year. Calculate
   (*i*) the rate of depreciation per annum.
   (*ii*) the original cost of the machine.
   (*iii*) the scrap value of the machine, if the estimated useful life is 20 years. (Use log tables)
7. A piece of machinery costing ₹25000 is expected to have a useful life of 4 years and a scrap value of zero. Find the annual depreciation charge using the sum of the years digits method.

Answers

1. ₹877.50 2. ₹20577 3. 5% p.a. 4. (*i*) ₹493125 (*ii*) ₹161587.20
5. ₹21725 6. (*i*) 5% p.a. (*ii*) ₹16000 (*iii*) ₹5729.60
7. ₹10000, ₹7500, ₹5000, ₹2500

MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options in questions (1 to 12):

1. ₹100 shares of a company are selling at ₹80. If the company is paying a dividend of 12%, then the rate of return is

   (*a*) 10% (*b*) 12% (*c*) 15% (*d*) 18%

2. Suman purchased 4000 shares of ₹100 each at a premium of ₹25. If company declares a dividend of 10%, then her rate of return is

   (*a*) 8% (*b*) 10% (*c*) 12% (*d*) 15%

3. Rohan invested ₹300000 in a fund for two years. At the end of two years the investment was worth ₹327000. Rohan's rate of return is

   (*a*) 6% (*b*) 7% (*c*) 8% (*d*) 9%

4. Vanshika invested ₹135000 on ₹100 shares at a discount of ₹10 paying 12% dividend. At the end of the year she sells the shares at a premium of ₹20, her rate of return is

   (*a*) 35.56% (*b*) 46.67% (*c*) 54.46% (*d*) 58.67%

5. The effective rate of return, which is equivalent to a declared rate of 12% compounded semiannually is

   (*a*) 11.86% (*b*) 11.98% (*c*) 12.36% (*d*) 12.54%

6. The declared rate of return compounded semiannually which is equivalent to 10.25%, effective rate of return, is

   (*a*) 10.13% (*b*) 10.05% (*c*) 10% (*d*) 9.89%

7. A person invested ₹180000 in a mutual fund in year 2016. If the value of mutual fund increased to ₹225000 in year 2020, then compound annual growth rate of his investment is [use $(1.25)^{1/4} = 1.057$)

   (*a*) 5.7% (*b*) 10.57% (*c*) 57% (*d*) none of these

8. An asset costing ₹15000 is expected to have a useful life of 5 years. If scrap value of the asset is ₹3000, then annual depreciation charge is

   (*a*) ₹3000 (*b*) ₹2400 (*c*) ₹2800 (*d*) ₹3600

9. A machine costing ₹C would reduce to ₹10000 in 7 years. If annual depreciation charge is ₹10000, then the value of C is

   (*a*) 80000 (*b*) 70000 (*c*) 60000 (*d*) none of these

10. A vehicle costing ₹125000 has scrap value of ₹25000. If annual depreciation charge is ₹12500, then useful life of the vehicle is

    (*a*) 4 years (*b*) 6 years (*c*) 8 years (*d*) 10 years

11. A newspaper printing machine costs ₹4,80,000 and estimated scrap value of ₹25000 at the end of its useful life of 10 years. What is its annual depreciation as per linear method?

    (*a*) ₹4550 (*b*) ₹45,500 (*c*) ₹50,500 (*d*) ₹61,500

12. An investment of ₹10,000 becomes ₹60,000 in 4 years, then the CAGR (compound annual growth rate) is given by

    (*a*) $\frac{\sqrt[4]{6}-1}{100}$ (*b*) $\frac{\sqrt[4]{6}+1}{100}$ (*c*) $(\sqrt[4]{6}-1)\times 100$ (*d*) $(\sqrt[4]{6}+1)\times 100$

Answers

1. (*c*)	**2.** (*a*)	**3.** (*d*)	**4.** (*b*)	**5.** (*c*)	**6.** (*c*)	**7.** (*a*)
8. (*b*)	**9.** (*a*)	**10.** (*c*)	**11.** (*b*)	**12.** (*c*)		

Assertion-Reason Type Questions (Solved)

These type of questions consist of two Statements.

Statement I is called Assertion (A) and Statement II is called Reason (R). Read the given Statements carefully and choose the correct answer from the options given below:

(a) *Both the Statements are true and Statement II is the correct explanation of Statement I.*

(b) *Both the Statements are true and Statement II is not the correct explanation of Statement I.*

(c) *Statement I is true, Statement II is false.*

(d) *Statement I is false, Statement II is true.*

1. ***Statement I:*** *A man invests ₹25000 in ₹100 shares available at ₹125. If the dividend paid by the company is 10%, then his rate of return is 8%.*

   Statement II: *Rate of return* $= \frac{\text{Net profit / loss}}{\text{cost of investment}} \times 100.$

2. ***Statement I:*** *Effective rate of return equivalent to a declared rate of 10% compounded quarterly is 10.38%. Given* $((1.025)^4 = 1.1038)$

   Statement II: *Effective rate of return* $= \left(1 + \frac{r}{100p}\right)^p$

   where r = annual rate of interest and p = number of compounding in a year.

3. ***Statement I:*** *If the nominal rate of interest is 12.5% and the inflation is 2%, then the effective rate of interest is 10.5%*

   Statement II: *If the interest is calculated only at the end of an year, the effective rate of interest is same as the nominal rate of interest.*

4. ***Statement I:*** *Mr Mishra invested ₹50,000 in a fund in year 2015. If the value of the fund in year 2020 is ₹75000, then compound annual growth rate of his investment is 8.45%. (Given* $(1.5)^{1/5} = 1.0845$*)*

   Statement II: $CAGR = \left(\frac{F.V.}{P.V.}\right)^{\frac{1}{n}} - 1.$

5. ***Statement I:*** *If the value of a machine depreciated by ₹290 during third year and by ₹232 during fourth year, then rate of depreciation per annum is 20%.*

   Statement II: *Depreciated value* $= P(1 - i)^n$

   where P = Present value; $i = \frac{r}{100}$, *r = constant rate of depreciation and n = number of years.*

Solutions to Assertion-Reason Type Questions

1. Market value of one share = ₹125

   Cost of investment = ₹25000

   $\therefore$ Number of shares bought $= \frac{25000}{125} = 200.$

   Dividend declared by the company = 10%

   It means dividend earned on each share $= 10 \times \frac{100}{100} = ₹10.$

   So, dividend earned on 200 shares = ₹(10 × 200) = ₹2000

   $\therefore$ Rate of return $= \frac{2000}{25000} \times 100\% = 8\%$

   $\therefore$ Statement I is true.

Also, Statement II is true and

Statement II is the correct explanation of Statement I.

$\therefore$ Option (a) is the correct answer.

2. Given $r = 10\%$, $p = 4$ (1 year = 4 quarters).

So, effective rate per rupee $= \left(1 + \frac{r}{100p}\right)^p - 1 = \left(1 + \frac{10}{400}\right)^4 - 1$

$= (1.025)^4 - 1 = 1.1038 - 1 = 0.1038$

$\therefore$ Effective rate of return $= (0.1038) \times 100\% = 10.38\%$

$\therefore$ Statement I is true.

Statement II is false.

$\therefore$ Option (c) is the correct answer.

3. Effective rate = Nominal rate – inflation

$= 12.5\% - 2\% = 10.5\%$

$\therefore$ Statement I is true.

Also, Statement II is true and

Statement II is not the correct explanation of Statement I.

$\therefore$ Option (b) is the correct answer.

4. Given, present value of investment = ₹50000,

final value of investment = ₹75000,

number of years (n) = 5.

So, CAGR $= \left(\frac{\text{F.V.}}{\text{P.V.}}\right)^{\frac{1}{n}} - 1 = \left(\frac{75000}{50000}\right)^{\frac{1}{5}} - 1$

$= (1.5)^{1/5} - 1 = 1.0845 - 1 = 0.0845.$

Hence, CAGR $= (0.0845) \times 100\% = 8.45\%$

$\therefore$ Statement I is true.

Also, Statement II is true and

Statement II is the correct explanation of Statement I.

$\therefore$ Option (a) is the correct answer.

5. Let the original cost of the machine be ₹P and the rate of depreciation be r% p.a., then the value of the machine after one year, two years, three years and four years are $P(1 - i)$, $P(1 - i)^2$, $P(1 - i)^3$ and $P(1 - i)^4$ respectively, where $i = \frac{r}{100}$.

According to given,

$P(1 - i)^2 - P(1 - i)^3 = 290$ and $P(1 - i)^3 - P(1 - i)^4 = 232$

$\Rightarrow$ $P(1 - i)^2 [1 - (1 - i)] = 290$ and $P(1 - i)^3 [1 - (1 - i)] = 232$

$\Rightarrow$ $P(1 - i)^2 i = 290$ ...(1) and $P(1 - i)^3 . i = 232$ ...(2)

Dividing (2) by (1), we get

$$1 - i = \frac{232}{290} \Rightarrow 1 - i = \frac{4}{5} \Rightarrow i = \frac{1}{5}$$

$$\Rightarrow \quad \frac{r}{100} = \frac{1}{5} \Rightarrow r = 20.$$

Hence, the rate of depreciation = 20% p.a.

$\therefore$ Statement I is true.

Also, Statement II is true and

Statement II is the correct explanation of Statement I.

$\therefore$ Option (a) is the correct answer.

Assertion-Reason Type Questions (Unsolved)

These type of questions consist of two Statements.

Statement I is called Assertion (A) and Statement II is called Reason (R). Read the given Statements carefully and choose the correct answer from the options given below:

(a) Both the Statements are true and Statement II is the correct explanation of Statement I.

(b) Both the Statements are true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

1. **Statement I:** A person invested ₹10000 in a fund for one year. If at the end of the year, the investment was worth ₹10500, then rate of return is 5%.

   Statement II: Rate of return $= \dfrac{\text{Net profit/loss}}{\text{Cost of investment}} \times 100.$

2. **Statement I:** Mr Verma took a loan of ₹4000 for 6 months. Lender deducts ₹500 as interest while lending. The effective rate of interest is 30% p.a.

   Statement II: Effective rate of interest $= \left(1+\dfrac{r}{200}\right)^2 - 1$ payable half yearly

3. **Statement I:** The declared rate of return compounded semi-annually is 8% which is equivalent to 8.16% p.a. effective rate of return.

   Statement II: Effective rate of interest $= \left(1+\dfrac{r}{200}\right)^2$ payable half-yearly.

4. **Statement I:** A sofaset costing ₹40000 has a useful life of 10 years. If annual depreciation is ₹3500, then its scrap value using linear method is ₹6000.

   Statement II: The formula of calculating annual depreciation by linear method is

   annual depreciation $= \dfrac{\text{original cost-scrap value}}{\text{useful life in years}}$.

5. **Statement I:** Nominal rate of return is the total amount of money earned by an investment without considering various expenses.

   Statement II: Actual rate of return can be calculated by subtracting the various expenses from the total money earned.

6. **Statement I:** Compound annual growth rate (CAGR) is calculated by the final and beginning value of an investment.

   Statement II: Compound annual growth rate (CAGR) describes the effect of compounding.

Answers

1. (a) 2. (d) 3. (c) 4. (d) 5. (b) 6. (b)