

Probability Distribution

1. Binomial Distribution

Definition: Fixed number of trials, two outcomes, independent trials, constant probability p .

$$q = 1 - p$$

Random Variable: $X \sim B(n, p)$

Probability Formula

$$P(X = r) = {}^nC_r \times p^r \times q^{(n-r)}$$

$${}^nC_r = n! / [r!(n-r)!]$$

Mean and Variance

$$\text{Mean } (\mu) = np$$

$$\text{Variance } (\sigma^2) = npq$$

$$\text{Standard Deviation } (\sigma) = \sqrt{npq}$$

Important Results

- Exactly r successes: $P(X=r) = {}^nC_r \cdot p^r \cdot q^{(n-r)}$
- No success: $P(X=0) = q^n$
- All successes: $P(X=n) = p^n$
- At least one success: $P(X \geq 1) = 1 - q^n$
- At least r successes: $\sum_{k=r}^n {}^nC_k p^k q^{(n-k)}$
- At most r successes: $\sum_{k=0}^r {}^nC_k p^k q^{(n-k)}$

Memory Trick

BINOM = Binary outcomes, Independent trials, Number fixed, One probability, Mean= np

2. Poisson Distribution

Limiting form of Binomial distribution used for rare events.

Conditions: $n \rightarrow \infty$, $p \rightarrow 0$, $np = \lambda$ (finite)

Random Variable: $X \sim P(\lambda)$

Probability Function

$$P(X=r) = e^{-\lambda} \lambda^r / r!$$

Mean and Variance

$$\text{Mean } (\mu) = \lambda$$

$$\text{Variance } (\sigma^2) = \lambda$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\lambda}$$

Recurrence Formula

$$P(r+1) = [\lambda / (r+1)] P(r)$$

Important Results

- $P(0) = e^{-\lambda}$
- $P(r) = e^{-\lambda} \lambda^r / r!$
- $P(X \geq 1) = 1 - e^{-\lambda}$

When to Use

- Rare events
- Very large n
- Very small p
- Independent occurrences

Binomial vs Poisson

Binomial	Poisson
Parameters: n, p	Parameter: λ
Mean = np	Mean = λ
Variance = npq	Variance = λ
Fixed trials	Rare events

Example 5

Q: A die is thrown 6 times. If getting an odd number is a success, find:

- (i) $P(5 \text{ successes})$
- (ii) $P(\text{at most 5 successes})$

Answer:

$$n = 6, p = 1/2, q = 1/2$$

$$(i) P(5) = {}^6C_5 (1/2)^6 = 3/32$$

$$(ii) P(X \leq 5) = 1 - P(6)$$

$$= 1 - (1/2)^6$$

$$= 63/64$$

Q: 5% of items are defective. Find the probability that in a sample of 10 items, not more than one item is defective.

Answer:

$$p = 1/20, q = 19/20, n = 10$$

$$P(X \leq 1)$$

$$= P(0) + P(1)$$

$$= (19/20)^{10} + 10(1/20)(19/20)^9$$

$$= (29/20)(19/20)^9$$

Five cards are drawn with replacement.

Given:

$$p(\text{spade}) = 1/4, q = 3/4, n = 5$$

(i) All five spades:

$$P(5) = (1/4)^5$$

(ii) Only three spades:

$$P(3) = {}^5C_3 (1/4)^3 (3/4)^2$$

$$= 90(1/4)^5$$

(iii) No spade:

$$P(0) = (3/4)^5$$

Exercise 9.3

- 1. A coin is tossed 5 times. Find the probability of getting (i) at least 4 heads (ii) at most 4 heads.**
- 2. A fair coin is tossed 6 times. What is the probability of getting exactly 4 heads? At least 4 heads?**
- 3. If X denotes the number of heads in a single toss of 4 fair coins, then find (i) $P(X = 3)$ (ii) $P(X < 2)$ (iii) $P(X \leq 2)$ (iv) $P(1 < X \leq 3)$.**
- 4. Ten coins are tossed. What is the probability of getting at least 8 heads?**
- 5. Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are of diamond.**
- 6. A coin is tossed four times. If X is the number of heads observed, then find the probability distribution of X .**
- 7. Find the probability distribution of the number of successes in two tosses of a die when a success is defined as getting a number greater than 4.**
- 8. Find the probability distribution of number of doublets in three throws of a pair of dice.**

9. From a well-shuffled pack of 52 cards, 3 cards are drawn one by one with replacement. Find the probability distribution of number of queens.
10. An urn contains 5 white and 3 blue balls. If two balls are drawn one by one with replacement, then find the probability distribution of the number of white balls drawn.
11. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs drawn.
12. A die is thrown 5 times. Find the probability that an odd number will turn up (i) exactly 3 times (ii) at least 4 times (iii) maximum 3 times.
13. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.
14. A pair of dice is thrown 6 times. Getting a total of 7 on the two dice is considered a success. Find the probability of getting (i) at least 5 successes (ii) exactly 5 successes.
15. A box contains 100 bulbs, 10 are defective. If 5 bulbs are drawn successively with replacement, find the probability that (i) none is defective (ii) exactly two are defective.
16. A box contains 100 tickets, each bearing a different number from 1 to 100. If 5 tickets are drawn successively with replacement from the box, then find the probability that all tickets bear numbers divisible by 10.
17. If the chance that any of the five telephone lines is busy at an instant is 0.01, then what is the probability that all lines are busy?

Exercise 9.4

1. Find the mean of the Binomial distribution $B(8, 1/4)$.

- 2. In a binomial distribution, if $n = 20$ and $q = 0.6$, then find its mean.**
- 3. In a binomial distribution, if mean is 5 and variance is 4, then find the number of trials.**
- 4. If the mean of a binomial distribution is 20 and its standard deviation is 4, then find the value of p .**
- 5. Find the expectation of number of heads in 20 tosses of a fair coin.**
- 6. A die is tossed twice. A success is defined as getting an odd number on a random toss. Find the mean and variance of the number of successes.**
- 7. If X follows binomial distribution with parameters $n = 5$, p and $P(X = 2) = 9 P(X = 3)$, then find the value of p .**
- 8. Three cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the mean and the variance of the number of red cards drawn.**
- 9. A die is thrown 3 times. Let X be 'the number of two's seen', then find the probability distribution of X . Also find the expectation of X .**
- 10. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as 'a number greater than 4'. Also find the mean, variance and standard deviation of the distribution.**
- 11. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces drawn. Also find the mean and standard deviation of the distribution.**
- 12. A die is tossed three times. Getting an even number is considered a success. Find the binomial distribution. Also find the variance of the distribution.**
- 13. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Hence, find the mean and variance of the distribution.**
- 14. If X follows a binomial distribution with mean 3 and variance $3/2$, then find $P(X \leq 5)$.**

15. If the sum of the mean and the variance of a binomial distribution for 54 trials is 30, then find the distribution.