CLASS XII APPLIED MATHEMATICS

FINANCIAL MATHEMATICS

2. REVISION: ANNUITY

What is an Annuity?

An annuity is a sequence of equal payments made at equal intervals of time for a fixed number of periods.

🧠 **Basic idea**
Annuity = Equal payment + Equal time interval + Fixed number of payments.

Important Notation

Symbol	Meaning
P	Principal / Present Value
R	Regular payment / instalment
A	Accumulated Amount / Future Value
i	Rate of interest per period (decimal)
n	Number of payment periods
r%	Rate of interest per annum

Rule
Rate and number of periods must use the SAME time unit. Monthly payment → monthly rate and number of months.

Ordinary Annuity

Payments are made at the end of each period.

Future Value of an Annuity

Ask: “How much will my regular deposits become in the future?”

Hidden equation

Future Value
Future Value = Amount accumulated at the end.

Example 1

₹10,000 is deposited annually for 5 years at 8% p.a. Find the accumulated amount.

R = ₹10,000, i = 0.08, n = 5

A = ₹10,000 × [(1.08)⁵ − 1] / 0.08

A ≈ ₹58,666

Present Value of an Annuity

Ask: “What are those future payments worth TODAY?”

Hidden equation

Present Value

Present Value = Value TODAY of future payments.

Example 2

A person receives ₹20,000 annually for 5 years. If the rate is 10%, find the present value.

R = ₹20,000, i = 0.10, n = 5

P = ₹20,000 × [1 − (1.10)⁻⁵] / 0.10

P ≈ ₹75,816

3. PERPETUITY

Basic Understanding

A perpetuity is a series of equal payments made at regular intervals FOREVER.

Trick

Annuity = FINITE payments. Perpetuity = INFINITE payments.

It is an annuity in which the payments continue without an end.

Present Value of Perpetuity

Start with the Present Value of an annuity:

Hidden equation

For a perpetuity, n → ∞, so $(1 + i)^{-n} \to 0$.

Hidden equation

Perpetuity

Ask: “How much money is needed TODAY to generate R every period forever?”

Example 3

A perpetuity pays ₹15,000 every year at 6% p.a. Find its present value.

P = ₹15,000 / 0.06

P = ₹2,50,000

Important: A perpetuity has no final payment, so a normal finite “future value at the end” does not exist. Its key valuation is the value at a given date, commonly the present value.

4. SINKING FUND

Basic Understanding

Suppose a company needs ₹10 lakh after 5 years. Instead of arranging ₹10 lakh at the end, it deposits a fixed amount regularly into an interest-earning fund.

Trick

Sinking Fund = SAVE REGULARLY now to BUILD a TARGET amount later.

A sinking fund is a regular-deposit plan designed to accumulate a specified future amount.

Formula

The sinking-fund formula comes directly from the Future Value of an Annuity.

Hidden equation

Hidden equation

Concept Connection

Sinking Fund = Future Value of Annuity, but the question asks: “How much should I deposit regularly to reach a future target?”

Example 4

A company wants ₹5,00,000 in 5 years at 8% p.a. Find the annual deposit.

S = ₹5,00,000, i = 0.08, n = 5

R = [₹5,00,000 × 0.08] / [$(1.08)^5$ − 1]

R ≈ ₹85,228 per year

5. EMI — EQUATED MONTHLY INSTALMENT

Basic Understanding

A loan is repaid through equal instalments. When the instalments are monthly, they are called EMIs.

Trick

EMI = Equal Monthly Instalment. It is the regular payment used to repay a present loan.

Each EMI contains a Principal component + Interest component.

EMI Formula

Hidden equation

P = loan amount; i = monthly rate of interest; n = total number of monthly instalments.

Important Conversions

Hidden equation

Hidden equation

Trick
Annual → Monthly: divide the annual percentage rate by 12, then convert % to decimal. Years → Months: multiply by 12.

Example 5

Loan = ₹5,00,000; rate = 12% p.a.; time = 5 years.
i = 12/(12 × 100) = 0.01
n = 5 × 12 = 60
EMI = ₹5,00,000 × $[0.01(1.01)^{60} / ((1.01)^{60} - 1)]$
EMI ≈ ₹11,122 per month

Why does the EMI formula work?

A loan is received TODAY, while EMIs are paid in the FUTURE. Therefore:

Hidden equation

Using the Present Value of Annuity formula:

Hidden equation

Rearranging gives the EMI formula.

Connection
EMI is NOT a completely new idea. It is the Present Value of an Annuity applied to a loan.

6. CONCEPT MAP

Concept	Think of it as…	Main Formula
Annuity — Future Value	Regular deposits → amount in future	$A = R[(1+i)^n-1]/i$
Annuity — Present Value	Future payments → value today	$P = R[1-(1+i)^{-n}]/i$
Perpetuity	Equal payments → forever	$P = R/i$
Sinking Fund	Regular saving → future target	$R = Si/[(1+i)^n-1]$

EMI	Present loan → equal monthly repayments	$EMI = Pi(1+i)^n/[(1+i)^n-1]$

🧠 **One-Line Revision**
Annuity = finite payments | Perpetuity = infinite payments | Sinking Fund = build a future target | EMI = repay a present loan

7. QUICK FORMULA SHEET

Future Value of Annuity: Hidden equation

Present Value of Annuity: Hidden equation

Perpetuity: Hidden equation

Sinking Fund: Hidden equation

EMI: Hidden equation

8. CLASS PRACTICE

Annuity: Find the accumulated amount when ₹8,000 is deposited annually for 5 years at 7% p.a.

Annuity: Find the present value of ₹15,000 received annually for 4 years at 10% p.a.

Perpetuity: A perpetuity pays ₹18,000 annually at 6% p.a. Find its present value.

Sinking Fund: Find the annual deposit required to accumulate ₹8,00,000 in 6 years at 7% p.a.

EMI: Find the EMI on a ₹4,00,000 loan at 10% p.a. for 5 years.

Concept: Which generally has the lower EMI: a 5-year loan or a 10-year loan? Which usually results in more total interest? Explain.

9. Questions

1. An annuity has _____ payments, while a perpetuity has _____ payments.
2. Future Value asks: “How much will my regular deposits _____?”
3. Present Value asks: “What are future payments worth _____?”
4. A sinking fund is used to build a _____ amount in the future.
5. For monthly EMI, annual rate must be converted into a _____ rate.